Q10, Q11 Questions?

New Stuff up: §11.6

Ratio & Root Tests. Q11 Test up, shortly.

Factorials n^n stuff

n^{th} power stuff.

Conditional-vs-Absolute Convergence

Apr 10-8:58 AM

$S = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} \Longrightarrow a_n = \frac{(-1)^{n-1}}{\sqrt{n}} \Longrightarrow$

$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{\frac{1}{\sqrt{n+1}}}{\frac{1}{\sqrt{n}}} \right| = \frac{1}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{1} \xrightarrow{n \to \infty} 1$

Inconclusive

But we know it's alternating and

$a_n \xrightarrow{n \to \infty} 0$

Apr 10-9:12 AM

$\sum_{n=1}^{\infty} \frac{\sin(n)}{3^n} = \sum a_n \Longrightarrow$ #4 Fix Teacher's Blunder for Extra Credit.

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\sin(n+1)}{3^{n+1}}\cdot\frac{3^n}{\sin(n)}\right| = \left|\frac{\sin(n+1)}{\sin(n)}\cdot\frac{1}{3}\right|$

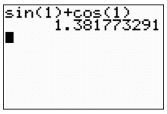
But Steve! $\sin\theta = \cos(\frac{\pi}{2}-\theta)$!

$\frac{\sin(n+1)}{\sin(n)} = \frac{\sin(n)\cos(1)+\sin(1)\cos(n)}{\sin(n)} = \sin(n)\cos(1)+\sin(1)\cos(\pi-n)$

$= \frac{\sin(n)\cos(1)+\sin(1)\sin(\pi-n)}{\sin(n)}$ $\sin\pi\cos n \boxed{-\sin n\cos\pi}$

$= \frac{\sin(n)\cos(1)+\sin(1)(\sin(n))}{\sin(n)} = \frac{\cancel{\sin(n)}\,[\cos(1)+\sin(1)]}{\cancel{\sin(n)}}$

$= \frac{\cos(1)+\sin(1)}{1} \approx 1.381773291$



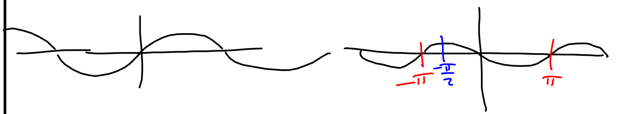
So $\left|\frac{a_{n+1}}{a_n}\right| \xrightarrow{n\to\infty} \frac{1.38\ldots}{3} < 1$

Cofunction identity

$\cos(\theta) = \sin(\frac{\pi}{2}-\theta)$ $\pi-\theta = -(\theta-\pi)$

$\sin(\theta) = \cos(\frac{\pi}{2}-\theta)$

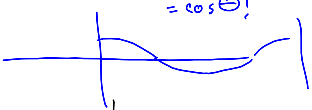
$\sin(\theta)$ $\sin(-\theta)$

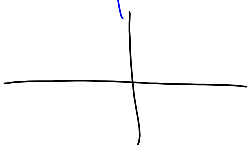


Keenan says Mills ~~is~~ is an idiot, b/c he messed-up cofunction identity

It's fixed, now.

$\sin(\frac{\pi}{2}-\theta) = \sin(-(\theta-\frac{\pi}{2}))$
$= \cos\theta$!





$$\sum_{n=1}^{\infty} \frac{(-3)^n}{(2n+1)!} = \sum a_n \implies$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-3)^{n+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{(-3)^n}\right| = \left|\frac{3 \cdot \cancel{(2n+1)} \cdot \cancel{(2(n-1)+1)} \cdots}{\underbrace{(2(n+1)+1)}_{2n+3} \cdot \cancel{(2n+1)!}}\right|$$

$$= \left|\frac{3}{2n+3}\right| \xrightarrow{n \to \infty} 0 \implies \sum a_n \text{ converges absolutely}$$

$\sum_{n=2}^{\infty} 3\left(1+\frac{1}{n}\right)^{n^2}$ Root TEST

$\sqrt[n]{3\left(1+\frac{1}{n}\right)^{n^2}} = 3^{\frac{1}{n}} \sqrt[n]{\left(1+\frac{1}{n}\right)^{n^2}}$

$= 3^{\frac{1}{n}} \left(1+\frac{1}{n}\right)^{\frac{n^2}{n}} = 3^{\frac{1}{n}} \underbrace{\left(1+\frac{1}{n}\right)^n}_{\to e^*} \xrightarrow{n\to\infty} 1 \cdot e > 1$

(* To see this, do logarithmic differentiation trick

$y = f(x)^{g(x)} \Rightarrow \ln y = \ln\left(f(x)^{g(x)}\right) = g(x) \ln(f(x))$)

$\boxed{\text{Diverges.}}$ By the root test.

In the rest of C II, Ratio Test is used the most.