Integral Test - Tough Integrals?

Comparison Test - Bonus

Limit Comparison Test - Sledgehammer

Test 3's in 80's & 100's

Test 3's in high 40's

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$

$$x\sqrt{x^2-5} = \sqrt{x^3-5x}$$

$$x\sqrt{x^2-5} = (x^2)^{\frac{1}{2}}(x^2-5)^{\frac{1}{2}} = \left(x^2(x^2-5)\right)^{\frac{1}{2}}$$

$$(ab)^c = a^c b^c$$

$$\sum \frac{n^2}{\sqrt{n^5-1}} \qquad \frac{n^2}{n^{\frac{5}{2}}} = \frac{1}{n^{\frac{1}{2}}}$$

Limit Comparison. Book way's good for these.

$$\frac{a_n}{b_n} = \frac{\frac{n^2}{\sqrt{n^5-1}}}{\frac{1}{n^{1/2}}} = \frac{n^2}{(n^5-1)^{1/2}} \cdot \frac{n^{\frac{1}{2}}}{1} = \frac{n^{\frac{5}{2}}}{\left(n^5\left(1-\frac{1}{n^5}\right)\right)^{\frac{1}{2}}}$$

$$= \frac{n^{\frac{5}{2}}}{(n^5)^{\frac{1}{2}}\left(1-\frac{1}{n^5}\right)^{\frac{1}{2}}} = \frac{n^{\frac{5}{2}}}{n^{\frac{5}{2}}\left(1-\frac{1}{n^5}\right)^{\frac{1}{2}}} = \frac{1}{\left(1-\frac{1}{n^5}\right)^{\frac{1}{2}}} \xrightarrow{n\to\infty} 1,$$

so $\sum a_n$ & $\sum b_n$ have same convergence properties, & $\sum b_n = \sum \frac{1}{n^{\frac{1}{2}}}$ diverges by $p = \frac{1}{2} < 1$ - test.

$2\pi\int_0^3 (y^2+2)\sqrt{4y^2+1}\,dy$

WebAssign Tests are auto-graded...

This means I can offer re-takes, without much trouble.

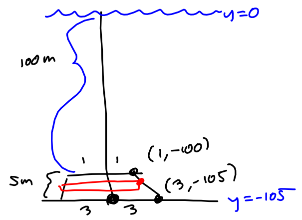
11.4 toughie Find $p \ni \sum_{n=2}^{\infty} \frac{1}{n^p \ln(n)}$ converges.

Lexicon: Identify variables in words & units

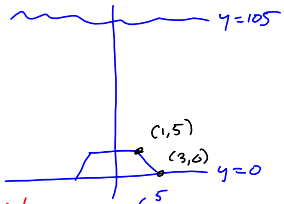
P = Population in # of trout = P(t) as a function of

t = time in years (# of years)

Pressure on a plate.



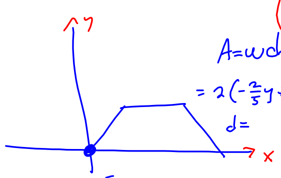
$\int_{-105}^{-100}$



$\int_{0}^{5}$

$\rho g A d$

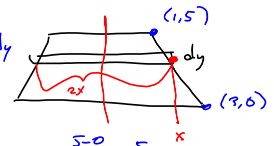
$\left(\frac{1000 \text{ kg}}{m^3}\right)\left(9.8 \frac{m}{s^2}\right)$



$A = w\,dy$

$= 2\left(-\frac{2}{5}y+3\right)dy$

$d =$



$F = 9800 \int_0^5 2\left(-\frac{2}{5}y+3\right)(105-y)\,dy$

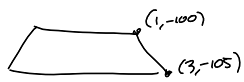
$m = \frac{5-0}{1-3} = -\frac{5}{2}$

$y = -\frac{5}{2}(x-3)+0$

$-\frac{2}{5}y = x-3$

$x = -\frac{2}{5}y+3$

$F = 9800 \int_{-105}^{-100} \left(-\frac{2}{5}y-39\right)(-y)\,dy$



$m = \frac{-100-(-105)}{1-3} = \frac{5}{-2}$

$y = -\frac{5}{2}(x-3)-105$

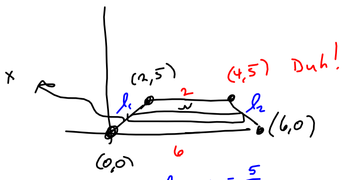
$y+105 = -\frac{5}{2}(x-3)$

$-\frac{2}{5}y - 42 = x-3$

$-\frac{2}{5}y - 39$

$-\frac{2}{5}(100) = -2(21) = -42$

unfortunate choice of coords:



l_1: $m = \frac{5}{2}$

$y_1 = \frac{5}{2}(x-0)+0 = \frac{5}{2}x$

$x = \frac{2}{5}y$

$w = x_R - x_L$

$= -\frac{2}{5}y + 6 - \frac{2}{5}y$

$= -\frac{4}{5}y + 6$

l_2: $m = -\frac{5}{2}$

$y_2 = -\frac{5}{2}(x-6)+0$

$y = -\frac{5}{2}x + 15$

$y - 15 = -\frac{5}{2}x$

$-\frac{2}{5}y + 6 = x$

$p^2+2p = t^3-t$ Solve for p

$p^2+2p+1 = t^3-t+1$

$(p+1)^2 = t^2-t+1$

$p+1 = \pm\sqrt{t^2-t+1}$

$p = 1\pm\sqrt{t^2-t+1}$

$2xy' + y = 2\sqrt{x}$ $\qquad$ $y' + P(x)y = Q(x)$

Want Integrating Factor.

$$y' + \frac{1}{2x}y = \frac{1}{2x}\cdot 2x^{\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

$I(x) = e^{\int P(x)dx}$, where $P(x) = \frac{1}{2x}$

$$\int P(x)dx = \int \frac{dx}{2x} = \frac{1}{2}\int \frac{dx}{x} = \frac{1}{2}\left[\ln(x) + \hat{\hat{C}}\right] = \ln\left(x^{\frac{1}{2}}\right) + \frac{1}{2}\hat{\hat{C}}$$

$$= \ln\sqrt{x} + \hat{C}$$

$$e^{\int P(x)dx} = e^{\ln\sqrt{x} + \hat{C}} = e^{\ln\sqrt{x}}e^{\hat{C}} = Ce^{\ln\sqrt{x}} = C\sqrt{x}$$

For our purposes, $C = 1$ is nicest.

$I(x) = \sqrt{x}$

$$\int \frac{dx}{2x} = \ln(2x) + C$$

Drop "| |"

$$\int \frac{du}{u} = \ln|u| + C$$

$$\frac{1}{2}\int \frac{2dx}{2x} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + \frac{1}{2}\hat{\hat{C}} = \frac{1}{2}\ln(2x) + \hat{C}$$

where $\hat{C} = \frac{1}{2}\hat{\hat{C}}$

$$= \frac{1}{2}\ln 2 + \frac{1}{2}\ln(x) + \hat{C}$$

$$= \frac{1}{2}\ln x + C, \text{ where } C = \hat{C} + \frac{1}{2}\ln 2$$

$$= \ln(\sqrt{x}) + C$$

$(ab)^c = a^c b^c$

$\boxed{a^b a^c = a^{b+c}}$

$a\ln b = \ln(b^a) \Longrightarrow I(x) = e^{\ln\sqrt{x} + C} = e^{\ln\sqrt{x}}e^C = (\sqrt{x})K = K\sqrt{x}$

where $K = e^C$

$\ln(ab) = \ln a + \ln b$

Choose $K = 1$

$I(x) = \sqrt{x}$

$a^{bc} = (a^b)^c = (a^c)^b$

"eigenvalue"

$\frac{d}{dx}$ is a linear operator.

$\frac{d}{dx}\left[c_1 f + c_2 g\right] = c_1 \frac{df}{dx} + c_2 \frac{dg}{dx}$

& $\frac{d}{dx}[0] = 0$

$y' = ky$ is like saying "k is an eigenvalue of y under the action of the linear operator $\frac{d}{dx}$"

$c, d \in \mathbb{R}$

f, g cnts

$\int (cf + dg) = c\int f + d\int g$

$A\bar{x} = \lambda \bar{x}$

A is matrix

$\bar{x}$ is vector (eigenvector)

λ is real # (eigenvalue)

$[A - \lambda I \mid \bar{0}]$

$|A - \lambda I| \overset{SET}{=} 0$

$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix}$

It turns out that $\{f \mid f: \mathbb{R} \xrightarrow{cnts} \mathbb{R}\}$ is an infinite-dimensional vector space.

A Basis for $C(\mathbb{R})$:

$1, t, t^2, t^3, \dots, t^n, \dots$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$e^x \approx \sum_{n=0}^{M} \frac{x^n}{n!}$

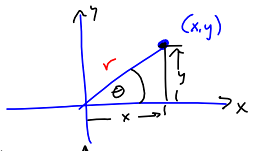
$\{\sin(mx) + \cos(mx) \mid m = 0, \dots, \infty\}$

$c_0 + c_1 \sin x + d_1 \cos x + c_2 \sin(2x) + d_2 \cos(2x) + \dots$

Maclaurin

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)\, x^n}{n!}$

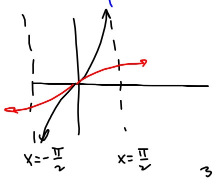
$\sum_{n=0}^{M} \frac{f^{(n)}(2)\,(x-2)^n}{n!}$



S 10.3

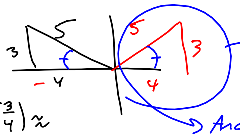
Given (x,y), $r = \sqrt{x^2+y^2}$

$\tan\theta = \frac{y}{x}$ &



$\theta = \arctan\left(\frac{y}{x}\right)$ if we're in the 1st or 4th quadrants

QI & QIV

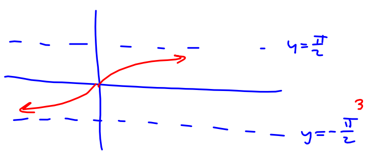


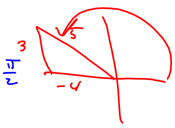
→ $\arctan\left(\frac{3}{4}\right)$

→ Arctan sees this.

$\arctan\left(\frac{3}{4}\right) \approx$

$\theta = 180° - \arctan\left(\frac{3}{4}\right)$





$(r,\theta) \longrightarrow (x,y)$

$x = r\cos\theta$

$y = r\sin\theta$