Having Microphone Issues.

Go into chat and ask me anything while I get this figured out.

Chapter 10 re-take is open 'til 4/11.

Today: Integral Test, Section 11.3

Next up: Comparison Tests.

Improper Integral Skills: HUGE for the next two sections.

$\sum_{k=2}^{\infty} \frac{2}{n(\ln(n))^p} \longleftrightarrow \int_2^{\infty} \frac{2\,dx}{x(\ln(x))^p} = 2\int_2^{\infty} (\ln(x))^{-p} \cdot \frac{1}{x}\,dx$

$= 2\int_2^{\infty} u^{-p}\,du$, where $u = \ln(x)$

$= \lim_{t\to\infty} \left[\frac{u^{-p+1}}{-p+1}\right]_2^t$

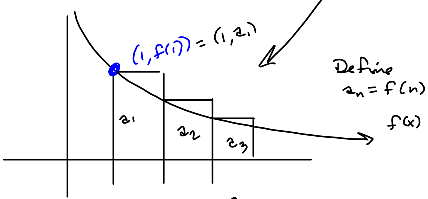
Need $p > 1$

$\sum_{k=1}^{\infty} a_k$

1 2 3

So, $\sum_{k=1}^{\infty} a_k \leq \int_0^{\infty} f(x)\,dx$

$\int_1^{\infty} f(x)\,dx \leq \sum_{k=1}^{\infty} a_k$



Define $a_n = f(n)$

This gives an upper & lower bound for $\sum a_k$, in terms of $\int_1^{\infty} f(x)\,dx$

Estimating $\sum_{k=1}^{\infty} a_k$ via

$S_n + \sum_{k=n+1}^{\infty} a_k = \sum_{k=1}^{n} a_k + \sum_{k=n+1}^{\infty} a_k$

The n-tail of the series gives us an error estimate on S using S_n

How good's our n^{th} partial sum? Only as good as $\sum_{n+1}^{\infty} a_k$ is small.

$\int_{n+2}^{\infty} f(x)\,dx \leq \sum_{k=n+1}^{\infty} a_k \leq \int_{n+1}^{\infty} f(x)\,dx$

A lot of improper integrals.

$\sum_{k=1}^{\infty} \frac{n^3}{n^8+1}$ Think: $\sum \frac{1}{n^5}$

Limit Comparison Test

$\sum \frac{n^3}{n^8+1}$'s convergence/divergence is the same as $\sum \frac{1}{n^5}$'s.

Direct Comparison – Bonus on Tests.

Show Convergence. $\frac{n^3}{n^8+1} < \frac{n^3}{n^8} = \frac{1}{n^5}$, so, if $\sum \frac{1}{n^5} \longrightarrow$

then $\sum \frac{n^3}{n^8+1} \longrightarrow$.

Show Divergence $\sum \frac{n^3}{n^4+1}$ Thought process: $\sum \frac{1}{n}$

Need $a_n > b_n$ & $\sum b_n \not\longrightarrow$

Assumptions: $a_n > 0 \ \forall n > M$, $a_{n+1} < a_n \ \forall n > N$

The old trick

only need this for Direct Comparisons

$\frac{n^3}{n^4+1} > \frac{n^3}{n^4+\frac{1}{2}n^4} = \frac{n^3}{\frac{3}{2}n^4} = \frac{2}{3}\left(\frac{1}{n}\right)$

11.4 stuff

$\frac{n^3}{n^4-1} > \frac{n^3}{n^4} = \frac{1}{n}$

It's necessary that $a_n \xrightarrow{n\to\infty} 0$ for $\sum_{n=1}^{\infty} a_n$ to converge, but it's not sufficient.

Limit Comparison Test : Making your intuition explicit.

$\sum a_n = \sum \frac{n^3}{n^8 - 1}$ compares to $\sum \frac{1}{n^5} = \sum b_n$

If $\lim \frac{a_n}{b_n} = c \in \mathbb{R}$

$\frac{n^8}{n^8} \xrightarrow{n \to \infty} 1$

$$\frac{\frac{n^3}{n^8 - 1}}{\frac{1}{n^5}} = \left(\frac{n^3}{n^8 - 1}\right)\left(\frac{n^5}{1}\right) = \frac{n^8}{n^8 - 1} = \frac{n^8}{n^8\left(1 - \frac{1}{n^8}\right)}$$

$$= \frac{1}{1 - \frac{1}{n^8}} \xrightarrow{n \to \infty} 1 \in \mathbb{R} \text{ so}$$

$\frac{1}{n^8} \to 0$

convergence props of $\sum a_n$ & $\sum b_n$ are same.

Isaac warns us about 11.2 #s 40-42: Dad Gum things don't work.

Don't waste time on things that don't work

or that you don't think you need for mastery.

TEC's are optional! Heck, they don't even work.

970-290-0550