$$a_{n+1} = 3 - \frac{1}{a_n}$$

11.1? 11.2?

If

$a_{n+1} > a_n$, then

$$\frac{1}{a_{n+1}} < \frac{1}{a_n}$$

$$-\frac{1}{a_{n+1}} > -\frac{1}{a_n}$$

$$a_{n+2} = 3 - \frac{1}{a_{n+1}} > 3 - \frac{1}{a_n} = a_{n+1}$$

Towards the end of 11.2

p-test for integrals

$$\int_1^\infty \frac{dx}{x^p} \quad \text{Need: } p>1$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \quad p>1$$

$\sum \frac{1}{n}$ is Harmonic Series.

Geometric Series.

$$R + Ri + Ri^2 + Ri^3 + \dots$$

$$\sum_{k=1}^{n} R(1+i)^{n-1}, \text{ where } R = \text{pmt } \&$$

$i = \frac{r}{m}$, where

$r =$ interest rate (Annual)

$\&$ $m =$ periods per year.

Savings Acct:

$$A = P(1+i)^{mt}$$

$$R(1+i)^{n-1} + R(1+i)^{n-2} + \dots + R(1+i)^2 + R(1+i) + R$$

$$S_n = \sum_{k=1}^{n} ar^{k-1} = n^{\text{th}} \text{ partial sum}$$

970-290-0550

$$S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$-\,rS_n = \quad ar + ar^2 + \dots + ar^{n-1} + ar^n$$

$$S_n - rS_n = (1-r)S_n = a - ar^n$$

$$S = \frac{a - ar^n}{1-r} = a\left(\frac{1-r^n}{1-r}\right)$$

Closed-form expression for the n^{th} partial sum.

$$\text{Annuity} = R\left[\frac{1-(1+i)^n}{1-(1+i)}\right]$$

$$= R\left[\frac{(1+i)^n - 1}{i}\right] = \text{Future Value of an Annuity.}$$

$$S_n = a\left(\frac{1-r^n}{1-r}\right) \xrightarrow{n\to\infty} a\left(\frac{1}{1-r}\right) \text{ if } \boxed{-1<r<1}$$

See #31

$$\sum_{n=1}^{\infty} \frac{(x-2)^{n-1}}{5^{n-1}} = \sum_{n=1}^{\infty} \left(\frac{x-2}{5}\right)^{n-1}$$

Need $-1 < \frac{x-2}{5} < 1$

$$-5 < x-2 < 5$$

$$\boxed{-3 < x < 7}$$

Radius of convergence: $r = 5$

Interval of convergence:

$I = (-3, 7)$

Telescoping Series

$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges/diverges "like" $\sum \frac{1}{n^2}$

Trick: Partial Fractions!

$$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$1 = A(n+1) + Bn$

$n=-1$ $\boxed{1 = -B}$ $n=0$ $\boxed{A=1}$

$$\sum \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S_n = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1}$$

$$= 1 - \frac{1}{n+1} \xrightarrow{n\to\infty} 1 = \sum \frac{1}{n(n+1)}$$