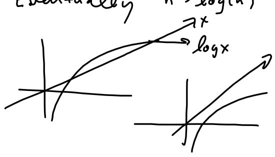
WebAssign? See Announcements in D2L (online.aims.edu).

Review Your Test? 2 attempts discussion... Doesn't quite work the way I hoped, so I'll just make another copy of the test, and let you take THAT test.

Test re-takes for Chapter 10, this week!

Eventually $n > \log(n)$



$\frac{n}{\log(n)} \xrightarrow{n\to\infty} \infty$

$\frac{x}{\log(x)} \xrightarrow{x\to\infty} \infty$

$\int_1^\infty f(x)\,dx \longleftrightarrow \sum_{k=1}^{\infty} f(k)$

$n! > n^2, n^5, n^{257}$

$5! = 5\cdot4\cdot3\cdot2\cdot1$

$n^n > n!$

$f > g \Rightarrow$

$\sum f > \sum g$

Mathematical Induction

Want to show a statement A is true $\forall n \in \mathbb{N}$.

Show it holds for $n=1$ (or 2 or 3...)

Show it holds for $n+1$ when it holds for n.

$\boxed{E}$ Recall $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

Proof: $\sum_{k=1}^{1} k = 1 \stackrel{?}{=} \frac{1(1+1)}{2} = 1$ ✓ $A(1)$ holds.

Assume $A(n)$ holds for some $n \geq 1$. We show $A(n+1)$ holds:

$A(n+1)$: $\sum_{k=1}^{n+1} k = \frac{(n+1)(n+1+1)}{2} = \frac{(n+1)(n+2)}{2}$

$$\sum_{k=1}^{n+1} k = \underbrace{1+2+3+\cdots+n}_{} + (n+1)$$

$$= \frac{n(n+1)}{2} + n+1 = \frac{n^2+n}{2} + \frac{2(n+1)}{2}$$

$$= \frac{n^2+n+2n+2}{2} = \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2} \Longrightarrow$$

$A(n+1)$ holds $\Longrightarrow$ Done, by PMI. ∎

(PMI : Principle of mathematical induction)

$a_1 = 1$, $a_{n+1} = 3 - \frac{1}{a_n}$

is increasing, bdd below by 2 & above by 3.

$a_2 = 3 - \frac{1}{a_1} = 3 - \frac{1}{1} = 2 > a_1 = 1$

Suppose $a_{n+1} > a_n$ for some $n \geq 1$

Then $a_{n+1} = 3 - \frac{1}{a_n} > a_n$ ← I'm subtracting something smaller!

$a_{n+2} = 3 - \frac{1}{a_{n+1}} > 3 - \frac{1}{a_n} = a_{n+1}$

B/c $a_{n+1} > a_n$

$2 < 3$

$\frac{1}{2} > \frac{1}{3}$

Need $a_n > 0$, $a_n < 3$, $a_n > 2$

$\left\{ \begin{array}{l} a_{n+1} > a_n \\ \frac{1}{a_{n+1}} < \frac{1}{a_n} \end{array} \right.$

$a_n > 2 \; \forall n \geq 3$

$a_1 = 1$

$a_2 = 2$

$a_3 = 2.5$

Suppose $a_n > 2$

Then $a_{n+1} = 3 - \frac{1}{a_n} > 3 - \frac{1}{2} = 2.5 > 2$

$\Rightarrow A(n+1)$ holds $\Rightarrow$ ∎

Find $\lim_{n \to \infty} a_n$ (after proving it HAS a limit)

(Prove it's increasing sqnc. Monotone
Prove it's bounded above)

(Lebesgue) Monotone Convergence Theorem.

Observe: If $a_n \xrightarrow{n \to \infty} L$ then $a_{n+1} \xrightarrow{n \to \infty} L$

This means that

$a_n \longrightarrow 3 - \frac{1}{a_n}$

$L = 3 - \frac{1}{L}$ & solve for L!

LCD = L

$\frac{L^2}{L} - \frac{3L}{L} + \frac{1}{L} = 0$

$\frac{L^2 - 3L + 1}{L} = 0 \Rightarrow$

$L^2 - 3L + 1 = 0$

$\Rightarrow L^2 - 3L + \left(\frac{3}{2}\right)^2 - \frac{9}{4} + \frac{4}{4} = \left(L - \frac{3}{2}\right)^2 - \frac{5}{4} = 0$

$\left(L - \frac{3}{2}\right)^2 = \frac{5}{4}$

$L - \frac{3}{2} = \pm \frac{\sqrt{5}}{2} \Rightarrow L = \frac{3 \pm \sqrt{5}}{2} \Rightarrow L = \frac{3 + \sqrt{5}}{2}$