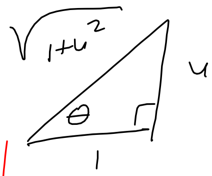
$$\int_0^2 \sqrt{1+e^{-2x}}\,dx$$

$u = e^{-x}$

$du = -e^{-x}\,dx = -u\,dx$

$dx = -\frac{du}{u}$

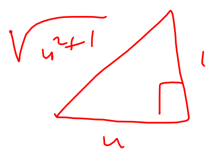
$$= \int_{x=0}^{x=2} \frac{-\sqrt{1+u^2}}{u}\,du$$



$u = \tan\theta$

$du = \sec\theta \tan d\theta\, d\theta$

$|\sec\theta|$



$u = \cot\theta$ is just as valid.

$$= \int_{x=0}^{x=2} -\frac{\sqrt{1+\tan^2\theta}\,\sec\theta\,\cancel{\tan\theta}}{\cancel{\tan\theta}}\,d\theta$$

$x = 0 \rightarrow$

$u = e^{-0} = 1$

$u = e^{-2}$

$u = \tan\theta$

$\frac{1}{e^2} = \tan\theta$

$\theta = \arctan\left(\frac{1}{e^2}\right)$

$u = 0 = \tan\theta$

$\theta = 0$

$$= -\int_{x=0 \to u=1}^{x=2 \to \frac{1}{e^2}=u} |\sec\theta|\sec\theta\,d\theta$$

$\sec\theta \geq 0$ on $[0, \frac{\pi}{2})$

$\supseteq [0, \frac{1}{e^2}]$

$(|\sec\theta| = \sec\theta)$

$$= -\int_0^{\arctan\left(\frac{1}{e^2}\right)} \sec^2\theta\,d\theta$$

$$= -\tan\theta\Big]_0^{\arctan\left(\frac{1}{e^2}\right)} = -\left[\tan\left(\arctan\left(e^{-2}\right)\right) - \tan(0)\right]$$

$$= -\left[\frac{1}{e^2} - 0\right] = -\frac{1}{e^2}$$

$$\sqrt{2} + \frac{\ln\left(\sqrt{2}-1\right)}{2} - \frac{\ln\left(1+\sqrt{2}\right)}{2} - \sqrt{1+e^{-4}} - \frac{\ln\left(\sqrt{1+e^{-4}}-1\right)}{2} + \frac{\ln\left(\sqrt{1+e^{-4}}+1\right)}{2}$$

$$= \sqrt{2} + \frac{1}{2}\ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right) - \sqrt{1+e^{-4}} + \frac{1}{2}\ln\left(\frac{\sqrt{1+e^{-4}}+1}{\sqrt{1+e^{-4}}-1}\right)$$

$$= \sqrt{2} - \sqrt{1+e^{-4}} + \frac{1}{2}\ln\left[\frac{(\sqrt{2}-1)\left(\sqrt{1+e^{-4}}+1\right)}{(\sqrt{2}+1)\left(\sqrt{1+e^{-4}}-1\right)}\right]$$

$$= \sqrt{2} - \sqrt{1+e^{-4}} + \frac{1}{2}\ln\left[\frac{\sqrt{2}\sqrt{1+e^{-4}} + \sqrt{2} - \sqrt{1+e^{-4}} - 1}{\sqrt{2}\sqrt{1+e^{-4}} - \sqrt{2} + \sqrt{1+e^{-4}} - 1}\right]$$

§'s 8.2 - 8.5 :

Setting things up > Evaluating Messy Integrals.

Ask about them when you get stuck.

Use your time wisely.

Chapter 9: Differential Equations! Knowing where something is from how it moves!