$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

$$a^b a^c = a^{b+c}$$

$$\ln(ab) = \ln a + \ln b$$

$$\frac{d}{dx}[\ln(f(x))] = \frac{1}{f(x)} \cdot f'(x) = \frac{f'(x)}{f(x)}$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{f'(x)\,dx}{f(x)} = \ln|f(x)| + C$$

$$\frac{d}{dx}\left[\ln\left(x^{\frac{4}{5}}\right)\right] = \frac{\frac{4}{5}x^{-\frac{1}{5}}}{x^{4/5}} = \frac{4}{5}\left[\frac{1}{x^{\frac{4}{5}}x^{\frac{1}{5}}}\right] = \frac{4}{5}\cdot\frac{1}{x}$$

$$\ln\left(x^{\frac{4}{5}}\right) = \frac{4}{5}\ln(x)$$

$$\frac{d}{dx}\left[\ln\left(x^{\frac{4}{5}}\right)\right] = \frac{4}{5}\frac{d}{dx}[\ln x] = \frac{4}{5}\cdot\frac{1}{x}$$

$$\left(a^b\right)^c = a^{bc}$$

$$\ln(x^n) = n\ln x$$

$$\left(2^3\right)^4 = 2^{12}$$

$$\int x^n f(x)\,dx \qquad u = x^n \qquad f(x)\,dx = dv, \text{ etc.}$$

$$du = nx^{n-1}$$

EXCEPT when $f(x) = \ln x$ $\left(\text{or } \log_3 x = \frac{\ln x}{\ln 3}\right)$

$$\int x^n \ln(x)\,dx \qquad u = \ln(x) \qquad dv = x^n dx$$

$$du = \frac{1}{x}dx \qquad v = \frac{1}{n+1}x^{n+1}$$

$$\int \frac{dx}{x(\ln(x))^4} = \int$$

u-sub.

$$u = \ln(x)$$

$$du = \frac{1}{x}dx$$

$$= \int \frac{1}{(\ln(x))^4}\cdot\frac{1}{x}\,dx$$

$$= \int u^{-4}\,du = \frac{u^{-3}}{-3} + C$$

$$4\ln x = \ln(x^4) \neq (\ln(x))^4$$

$$= -\frac{1}{3(\ln(x))^3} + C$$

Arc Length

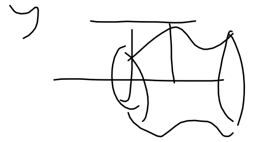
ds = arc length 'increment' $= \sqrt{1+f'(x)^2}\,dx$

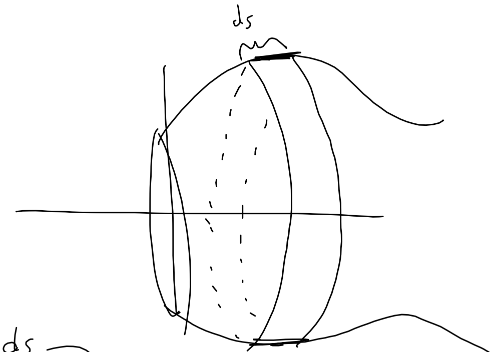
$L = \int_a^b ds$ when $x=g(y)$ instead of $y=g(x)$, then $\int_a^b ds = \int_a^b \sqrt{1+g'(y)^2}\,dy$

Surface area of a surface of revolution.

$A = 2\pi \int_c^d y\,ds$ for revolving about x-axis

radius is the distance from x-axis to the func.





About x-axis

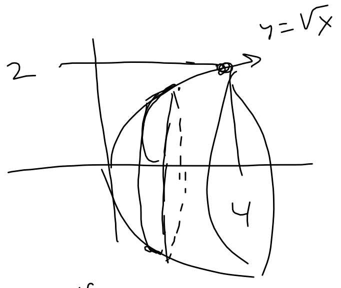
$\boxed{2\pi \int_a^b y\,ds} = 2\pi \int_a^b f(x)\,ds$ → when $y=g(x)$

$= 2\pi \int y\sqrt{1+g'(y)^2}\,dy$ when $x=g(y)$

$2\pi \int_c^d x\,ds$ about the y-axis

$2\pi \int_a^b x\sqrt{1+f'(x)^2}\,dx$

$2\pi \int_c^d g(y)\sqrt{1+g'(y)^2}\,dy$



$2\pi \int_0^4 \sqrt{x}\sqrt{1+\left(\frac{1}{2}x^{-\frac{1}{2}}\right)^2}\,dx$

$= 2\pi \int_0^2 y\sqrt{1+(2y)^2}\,dy$

$y = \sqrt{x}$

$g(y) = y^2 = x$

$g'(y) = 2y$

FORCE ON SUBMERGED PLATE

Exponential Growth $y = A_0 e^{kt}$

Logistic GROWTH $y = \dfrac{A}{B + Ce^{kt}}$

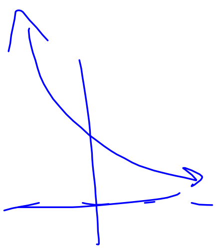
Probability Density Function

$$\int_{-\infty}^{\infty} P(x)\,dx = 1 \qquad P(x) \ge 0$$

opportunity for 'improper integrals

$P(x) = Ce^{-x}$ on $[0,\infty)$

$$C\int_0^{\infty} e^{-x}\,dx = -Ce^{-x}\Big]_0^{x\to\infty}$$



$y=0$

$$\xrightarrow{x\to\infty} \text{"} -Ce^{-\infty}\text{"} - (-Ce^{-0}) = C \overset{SET}{=} 1$$

Show that $y=e^{-x}$ is a probability density ~~funcn~~ function on $[0,\infty)$

CONTINUOUS PROBABILITY DISTRIBUTION.

(Excuse for an improper integral)

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{17x} = \lim_{x\to 0}(1+x)^{\frac{17}{x}}$$

$$\lim\left(\left(1+\frac{1}{x}\right)^{x}\right)^{17}$$

$$= \left(\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\right)^{17} = e^{17}$$

$e \equiv \lim_{x\to\infty}\left(\left(1+\frac{1}{x}\right)^{x}\right)$ is its definition.

$y = \left(1+\frac{1}{x}\right)^{17x}$ is a 1^{∞} situation

$$\ln(y) = \ln\left(\left(1+\frac{1}{x}\right)^{17x}\right) = 17x\ln\left(1+\frac{1}{x}\right) \xrightarrow{x\to\infty} \infty\cdot\ln(1)$$

$$= \infty\cdot 0$$

$$17x\ln\left(1+\frac{1}{x}\right) = \frac{17\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}$$

$$\lim_{x\to\infty}\left(\frac{17\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}\right) \overset{L'H}{=} \lim_{x\to\infty} 17\,\frac{\left(\frac{-\frac{1}{x^2}}{1+\frac{1}{x}}\right)}{-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{1}{1+\frac{1}{x}} = 1$$

So, $\lim_{x\to\infty}(\ln(y)) = 17$

$\Rightarrow y = e^{17}$

$$\frac{d}{dx}\left[\ln\left(1+\frac{1}{x}\right)\right] = \frac{-\frac{1}{x^2}}{1+\frac{1}{x}}$$

$$\frac{d}{dx}\left[1+\frac{1}{x}\right] = \frac{d}{dx}\left[1+x^{-1}\right] = -x^{-2} = -\frac{1}{x^2}$$

$$4y''' - 4y'' - 3y' + 3y = 0$$

$$(4D^3 - 4D^2 - 3D + 3)y = 0$$

$4D^3 - 4D^2 - 3D + 3 = 0$ is characteristic equation.

$$4D^2(D-1) - 3(D-1) = (D-1)(4D^2-3) = 0$$

$$D = 1 \qquad 4D^2 - 3 = 0$$

$$4D^2 = 3$$

$$D^2 = \frac{3}{4}$$

$$D = \pm\frac{\sqrt{3}}{2}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

1

1 1

$\boxed{1 \quad 2 \quad 1}$

1 3 3 1

$$\boxed{y = c_1 e^x + c_2 e^{\frac{\sqrt{3}}{2}x} + c_3 e^{-\frac{\sqrt{3}}{2}x}}$$

Separable Eq'ns

$$y' + ky = 0$$

$$y' = -ky$$

$$\frac{y'}{y} = -k$$

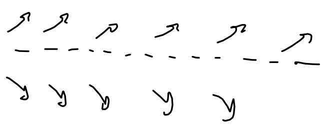
$$\frac{\frac{dy}{dx}}{y} = -k$$

$$\int \frac{dy}{y} = -k \int dx$$

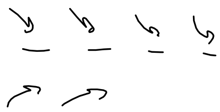
$$\ln y = -kx + C$$

$$e \qquad e$$

$$y = e^{-kx+C} = e^{C} e^{-kx} = Ce^{-kx}$$



- unstable equilibrium



- Stable equilibrium.

$$\frac{dP}{dt} = K(M-P)$$

$$\frac{\frac{dP}{dt}}{M-P} = K$$

$$\frac{dP}{M-P} = K\,dt \qquad u = M-P \qquad du = -\,dP$$

$$-\int \frac{-dP}{M-P} = -\int \frac{du}{u} = -\ln|u| + C$$

$$= -\ln|M-P| + C$$

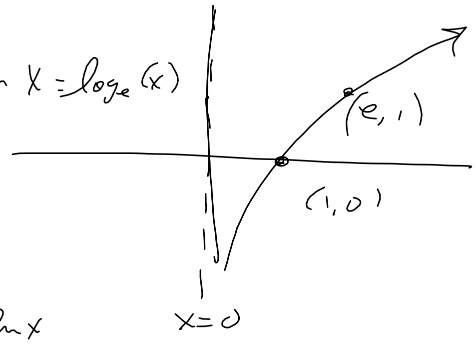
$-\ln(M-P)$

$-\ln(100-x)$

$= -\ln(-x+100)$

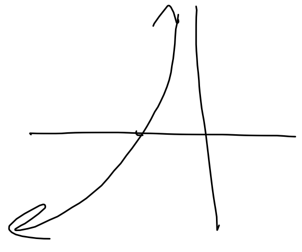
$= -\ln(-(x-100))$

$\ln x = \log_e(x)$

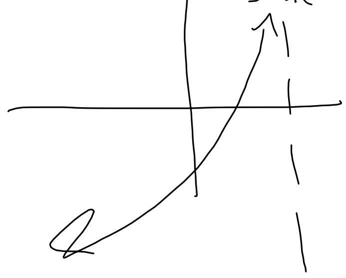


$-\ln x$

$-\ln(-x)$



$-\ln(-(x-100))$



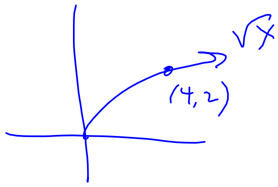
(0) $f(x) = \ln(x)$

(1) $-f(x) = -\ln(x)$ $\qquad y \longmapsto -y$

(2) $-f(-x) = -\ln(-x)$ $\longleftrightarrow$ $\qquad x \longmapsto -x$

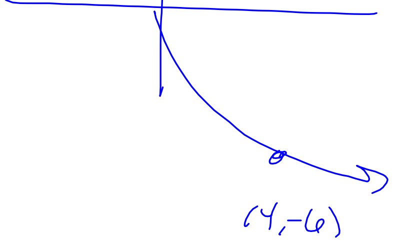
(3) $-f(-(x-100))$ $\qquad x \longmapsto x+100$

$g(x) = -3\sqrt{7x-42} + 11$



$-3\sqrt{x}$

$y \longmapsto -3y$

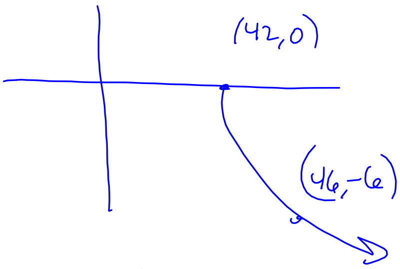


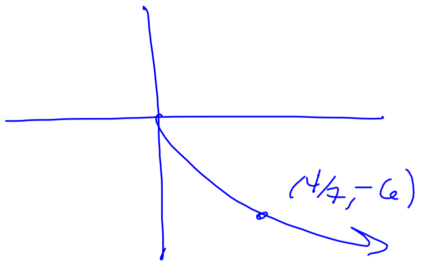
M1

$-3\sqrt{x-42}$ $\quad x \longmapsto x+42$

M2

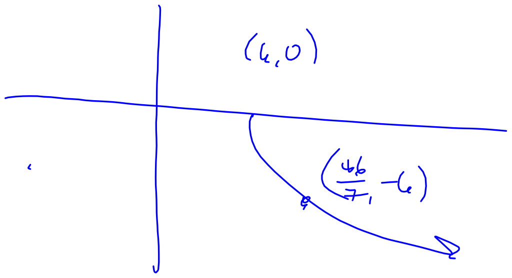
$-3\sqrt{7x}$ $\quad x \longmapsto \frac{1}{7}x$





M1

$-3\sqrt{7x-42}$ $\quad x \longmapsto \frac{x}{7}$



M2

$-3\sqrt{7(x-6)}$ $\quad x \longmapsto x+6$

$\frac{4}{7} + \frac{6}{1} \cdot \frac{7}{7} = \frac{4+42}{7} = \frac{46}{7}$