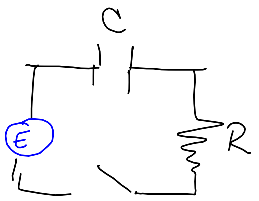
§9.5 #29



I = current

Q = charge

$I = \frac{dQ}{dt}$

By physics magic

$R = 5\,\Omega$

$C = .05$ Farad

$E = 60$ V

$Q(0) = 0$

$$RI + \frac{1}{C}Q = E(t)$$

$$R\frac{dQ}{dt} + \frac{1}{C}Q = E(t)$$

$$5Q' + \frac{1}{.05}Q = 60$$

$$5Q' + 20Q = 60$$

$$Q' + 4Q = 12$$ can be solved by integrating factor method.

$R(t) = 4 \Rightarrow$ i.f. $= e^{\int 4\,dt} = e^{4t + \hat{C}}$

$= e^{\hat{C}}e^{4t} = Ce^{4t}$

C doesn't matter, really, so assume C = 1 here.

$$\underbrace{e^{4t}Q' + 4e^{4t}Q}_{\left(e^{4t}Q\right)'} = 12e^{4t}$$

$$\left(e^{4t}Q\right)' = 12e^{4t}$$

$$e^{4t}Q(t) = \frac{12}{4}e^{4t} + \tilde{C} \quad \left(\div e^{4t}\right)$$

$$Q(t) = 3 + \tilde{C}e^{-4t}$$

Initial Condition: $Q(0) = 3 + \tilde{C}e^{-4(0)} = 3 + \tilde{C} = 0 \Rightarrow \tilde{C} = -3$, so

$$\boxed{Q(t) = 3 - 3e^{-4t}}$$

is the particular sol'n corresponding to the initial condition $Q(0) = 0$ that was given.

Treat as separable eq'n:

$$Q' + 4Q = 12 \Rightarrow$$

$$Q' = 12 - 4Q \Rightarrow$$

$$\frac{Q'}{12 - 4Q} = 1$$

$$\frac{1}{4}\int \frac{Q'}{3-Q}\,dt = \int dt \qquad u = 3-Q$$
$$du = -Q'dt$$

$$\Rightarrow -\frac{1}{4}\int \frac{-Q'dt}{3-Q} = t + \hat{c} \qquad \int \frac{du}{u} = \ln|u| + C$$

$$-\frac{1}{4}\ln|3-Q| = t + \hat{c}$$

$$\ln|3-Q| = -4t + c \qquad (c = -4\hat{c})$$

$$|3-Q| = e^{-4t+c} = \tilde{c}e^{-4t}$$

Assume $0 \le Q \le 3$:

$$-Q = -3 + Ce^{-4t}$$

$$Q = 3 - Ce^{-4t}$$

$$Q(0) = 0 \Rightarrow 3 - c = 0 \Rightarrow$$

$$c = 3 \Rightarrow$$

$$\boxed{Q(t) = 3 - 3e^{-4t}}$$

$$\int \frac{\ln x}{x^2}\,dx$$

$$\int (\ln x)\left(\frac{1}{x}\right)dx = \frac{1}{2}(\ln x)^2 + C \qquad \begin{pmatrix} u = \ln x \\ du = \frac{1}{x}dx \end{pmatrix}$$

But this?

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$ is Maclaurin Series for $f(x)$, assuming f is ∞'ly difbl

Taylor's $\sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!} (x-c)^k$

$e^x = f(x)$

$f(0) = 1$

$f'(0) = 1$

$f''(0) = 1$

$f^{(n)}(0) = 1$

So $e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$ $\qquad 0! = 1$

$= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5$

So $\int e^x dx = e^x + C$

$= \int \sum_{k=0}^{\infty} \frac{x^k}{k!} dx = \sum_{k=0}^{\infty} \frac{x^{k+1}}{(k+1)!}$ $+ C$

$\curvearrowright c = 1$ doesn't!

$(k+1)(k \cdot (k-1)(k-2)\cdots(3)(2)(1)) = (k+1)!$

$$\int \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) dx$$

$= C + x + \frac{1}{2}x^2 + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$

FACT These series converge QUICKLY to their $f(x)$.
So the 1st 4 or 5 terms is all you might need.

And so, MOST of what you want to do, Boils Down to working with Polynomials!