$(x-1)(x+5) = x^2+4x-5$

$D = \frac{d}{dx}$ $D^2 = \frac{d^2}{dx^2}$

$y'' + 4y' - 5y = 0$

"Characteristic Equation" $\rightarrow$ $(D^2+4D-5)y = 0$

Test Question for sure.

In a sense, $r = 1, -5$ are "eigenvalues."

$D^2+4D-5 = 0$

$(D+5)(D-1) = 0$

Roots of the Characteristic Equation

$D = -5, 1$ Now the cool part:

$\boxed{y = c_1e^{-5x} + c_2e^{x}}$ for any $c_1, c_2 \in \mathbb{R}$

The general solution.

Model Force.

$y' = -5c_1e^{-5x} + c_2e^{x}$

$y'' + 4y' - 5y =$ $y'' = 25c_1e^{-5x} + c_2e^{x}$

$= 25c_1e^{-5x} + c_2e^{x} + 4(-5c_1e^{-5x} + c_2e^{x}) - 5c_1e^{-5x} - 5c_2e^{x}$

$= (25-20-5)c_1e^{-5x} + (1+4-5)c_2e^{-x} = 0$ ✓

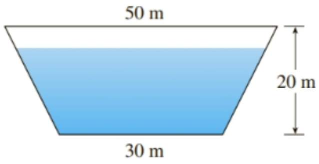
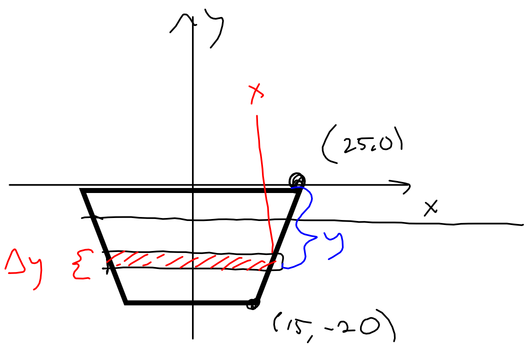


FIGURE 2

EXAMPLE 1 A dam has the shape of the trapezoid shown in Figure 2. The height is 20 m and the width is 50 m at the top and 30 m at the bottom. Find the force on the dam due to hydrostatic pressure if the water level is 4 m from the top of the dam.



$y=-4$ = Water level

Keenan

~~Isaac~~

$$\left(\frac{\text{Force}}{\text{unit area}}\right)(\text{Area})$$

$$= \left(\frac{\text{Force}}{m^2}\right)(x \cdot \Delta y \cdot 2)$$

$$(x_1, y_1) = (25, 0)$$

$$(x_2, y_2) = (15, -20)$$

$$m = \frac{-20-0}{15-25} = \frac{-20}{-10} = 2$$

$$y = 2(x-25)+0$$

$$y = 2x - 50$$

$$y + 50 = 2x$$

$$x = \tfrac{1}{2}(y+50)$$

$$\left(\frac{\text{Force}}{m^2}\right)\left(2 \cdot \tfrac{1}{2}(y+5)\right)\Delta y$$

$$= \frac{F}{m^2}(y+5)\Delta y$$

$$\vdots$$

$$\int_{-20}^{-4} \rho g D \,(y+50)\, dy$$

John

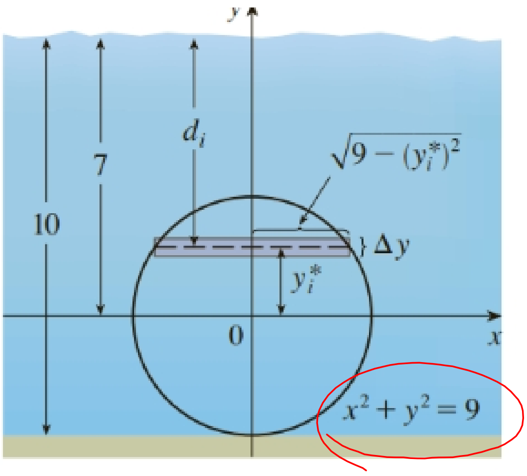
$$= \int_{-20}^{-4} \left(\frac{1000\,\text{kg}}{m^3}\right)\left(-\frac{9.8\,m}{s^2}\right)(-y-4)\,(y+50)\,dy$$

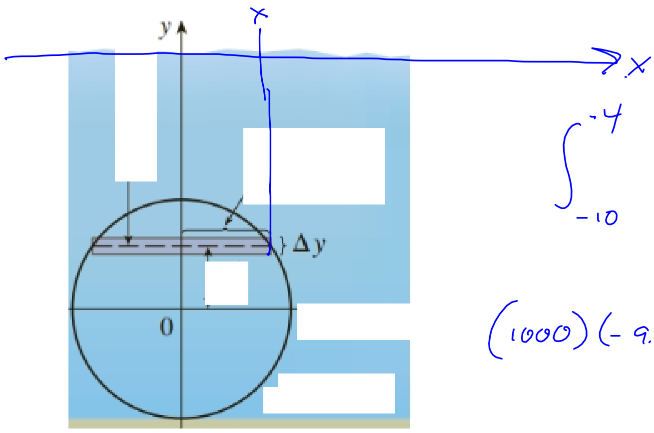
$$-(-15)-4 = 11$$

$$y = -4$$

$$D = 4$$

$$-(y+4)$$





$$\int_{-10}^{-4}$$

$$(1000)(-9.8)\int_{y=-10}^{-4}\left(2\sqrt{9-(y+7)^2}\right)(-y)\,dy$$

$u = y+7$

$du = dy$

$y = u-7$

Area: $x^2 + (y+7)^2 = 3^2$

$x^2 = 9-(y+7)^2$

$x = \pm\sqrt{9-(y+7)^2}$

Right $\frac{1}{2}$:

$x = \sqrt{9-(y+7)^2}$

Area $= 2x = 2\sqrt{9-(y+7)^2}$

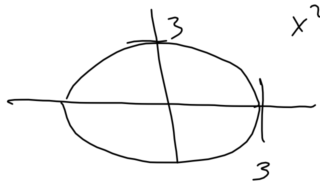
$$\underbrace{(1000)(-9.8)}_{-M}\int_{y=-10}^{-4}\left(2\sqrt{9-(y+7)^2}\right)(-y)\,dy$$

$x = \pm\sqrt{9-y^2}$ left & right half.

$u = y+7 \qquad du = dy$

$y = u-7$

$$2M\int_{y=-10}^{y=-4}\left(\sqrt{9-u^2}\right)(u-7)\,du$$



$x^2+y^2=3^2$

$y^2 = 9-x^2$

$y = \pm\sqrt{9-x^2}$

$$=2M\int_{y=-10}^{y=-4}\left(u\sqrt{9-u^2} - 7\sqrt{9-u^2}\right)du$$

$y=-4 \rightarrow u=3$

$y=-10 \rightarrow u=-3$

u-sub.

$\int_{-3}^{3}\sqrt{9-x^2}\,dx$ is area of top $\frac{1}{2}$ of circle.

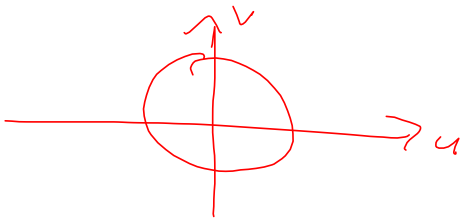
$u^2+v^2=3^2$

$v^2 = 9-u^2$

$v = \pm\sqrt{9-u^2}$

$$\int_{-3}^{3}\sqrt{9-u^2}\,du$$

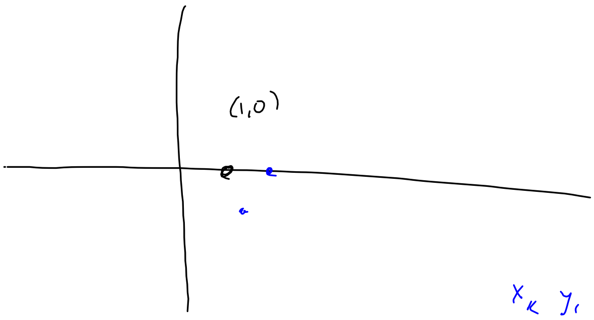
$\frac{1}{2}$ the area of a circle of radius 3.



$y' = y - 2x \qquad y(1) = 0 \qquad (x_0, y_0) = (1, 0)$

$\Delta x = h = .5$

$y'(1,0) = 0 - 2(1) = -2 = F(x_0, y_0)$



$y_1 = -2h = -2 \cdot .5 = -1$

$x_1 = x_0 + .5 = 1.5$

$x_k \quad y_k \quad F(x_k, y_k) \quad h$

$y_k = y_{k-1} + F(x_{k-1}, y_{k-1}) \cdot h$

$y_{k+1} = y_k + F(x_k, y_k) h$

$y = mx + b$

$y_2 = y_1 + m(x - x_1)$