$$\int_1^{\infty} \frac{dx}{x^2+x+10}$$ $p=2$-test

converges by direct comparison

$f(x) = \frac{1}{x^2}$

The comparison:

$$\int \frac{dx}{x^p}$$

$$\frac{1}{x^2+x+10} < \frac{1}{x^2}$$

$$\int \frac{dx}{x^2-x-10}$$ converges, by p-test

$$\frac{1}{x^2-x-10} > \frac{1}{x^2}$$ $x^2 > \frac{1}{2}x^2$

For x sufficiently large, $\frac{1}{2}x^2 > x$, $\frac{1}{4}x^2 > 10$

$$\frac{1}{x^2-x-10} < \frac{1}{x^2-\frac{1}{2}x^2-10} < \frac{1}{x^2-\frac{1}{2}x^2-\frac{1}{4}x^2}$$

$$= \frac{1}{\frac{1}{4}x^2} = \frac{4}{x^2}$$

& $\int \frac{4dx}{x^2}$ converges,

so $\int \frac{dx}{x^2-x-10}$ converges

$$\int \frac{dx}{x^{3/4}+x^{1/2}+1}$$ Diverges, by $p=\frac{3}{4}$ test

It's not easy to compare it to something SMALLER that diverges.

$$\int \frac{dx}{x^{3/4}-x^{1/2}-1}$$ $$\frac{1}{x^{3/4}-x^{1/2}-1} > \frac{1}{x^{3/4}}$$ & $\int_1^{\infty} \frac{dx}{x^{3/4}}$ Diverges,

$$\frac{1}{x^{3/4}+x^{1/2}+1} > \frac{1}{x^{3/4}+\frac{1}{4}x^{3/4}+\frac{1}{4}x^{3/4}} = \frac{1}{\frac{1}{2}x^{3/4}}$$ & $\int^{\infty} \frac{2dx}{x^{3/4}}$ Diverges.

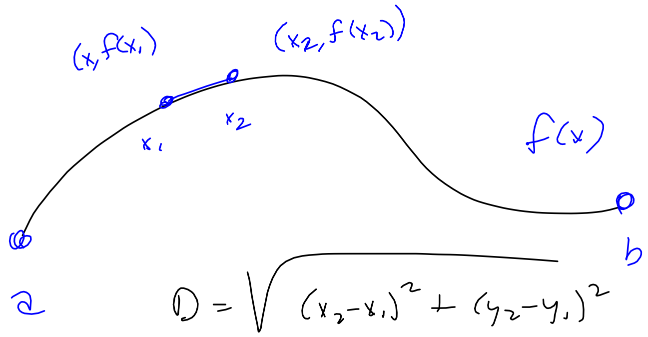
eventually, the $2x^{3/4} > x^{1/2}$

$x^2 > x$ eventually

$\frac{1}{100}x^2 > x$ eventually.

In Calc II the "limit comparison" just lets you say $\frac{1}{x^2+x+1}$ compares to $\frac{1}{x^2}$ in the limit.

DIRECT comparison is more subtle & is mainly bonus.



$$D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

$$= \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$= \sqrt{\Delta x^2 \left(1 + \left(\frac{\Delta y}{\Delta x}\right)^2\right)}$$

$$= \Delta x \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}$$

$$= \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x \xrightarrow{\Delta x \to 0}$$

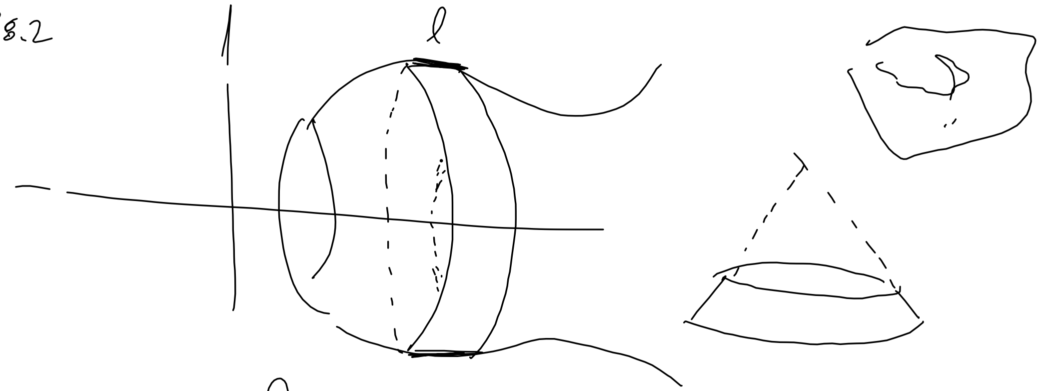
$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

Arclength $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\,\Delta x$

§8.1

$$= \int_a^b \sqrt{1 + f'(x)^2}\,dx$$

§8.2



$2\pi r \ell$

$$= 2\pi \underbrace{f(x)}_{\text{radius.}} \underbrace{\sqrt{1+(f'(x))^2}\, dx}_{\text{Arclength increment} = ds}$$

$2\pi \int y\, ds$ about x-axis $\to$ $2\pi \int f(x)\, ds$, $2\pi \int y\, ds$ *

$2\pi \int x\, ds$ about y-axis $\to$ $2\pi \int g(y)\, ds$, $2\pi \int x\, ds$ ☆

* when $x = g(y) \Rightarrow ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

☆ when $y = f(x) \Rightarrow ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

About x-axis : Instinct is $2\pi \int f(x)\sqrt{1+f'(x)^2}\, dx$

" y-axis : " " $2\pi \int g(y)\sqrt{1+g'(y)^2}\, dy$

18 §8.1

$f(x) = 1 - e^{-x}, \quad 0 \le x \le 2$

$f'(x) = e^{-x} \rightarrow (e^{-x})^2 = e^{-2x}$

$$s = \int_0^2 \sqrt{1 + e^{-2x}}\, dx$$

$$1 + e^{-2x} = \left(\frac{e^{2x}+1}{e^{2x}}\right)\left(\frac{e^{2x}-1}{e^{2x}-1}\right) = \frac{e^{4x}-1}{e^{4x}-e^{2x}}$$

$$\frac{\sqrt{e^{2x}+1}}{e^{x}}$$

$$\int \sqrt{1+e^{-2x}}\, dx$$

$u = e^{-x}$

$du = -e^{-x} dx \longrightarrow dx = \dfrac{du}{-e^{-x}}$

So $\left(\sqrt{1+e^{-2x}}\right)\left(-\dfrac{du}{e^{-x}}\right)$

$\cancel{\int \sqrt{1+e^{-2x}}}$ $\quad \displaystyle\int \left(\sqrt{1+u^2}\right)\left(\frac{-du}{u}\right)$

$$= \int \frac{\sqrt{1+u^2}}{u}\, du = \int \frac{\sqrt{1+w^2}}{w}\, dw$$

$u = \sqrt{1+w^2}$ $\qquad dv = \frac{1}{w} dw$

$du = \frac{1}{2}(1+w^2)^{-\frac{1}{2}}(2w)\, dw$ $\qquad v = \ln|w|$

$$uv - \int v\,du = \ln|w|\sqrt{1+w^2} - \int \ln|w| \frac{w}{\sqrt{1+w^2}}\,dw$$

$$w = e^{-x}$$

$$= \ln|e^{-x}|\sqrt{1+e^{-2x}} - \int \ln|e^{-x}| \frac{e^{-x}}{\sqrt{1+e^{-2x}}}\,dx$$

$$= -x\sqrt{1+e^{-2x}} - \int e^{-x}\frac{e^{-x}}{\sqrt{1+e^{-2x}}}\,dx = A - I_1$$

$$I_1 = \int \frac{e^{-2x}\,dx}{\sqrt{1+e^{-2x}}} \qquad u = 1+e^{-2x}, \quad du = -2e^{-2x}\,dx$$

$$= -\int \frac{-2e^{-2x}\,dx}{\sqrt{1+e^{-2x}}} = -\int \frac{du}{\sqrt{u}} = -\int u^{-\frac{1}{2}}\,du$$

$$= -2u^{\frac{1}{2}} + C = -2(1+e^{-2x})^{\frac{1}{2}} + C = I_1$$

So $I = A - I = -x\sqrt{1+e^{-2x}} + 2\sqrt{1+e^{-2x}} + C$

$$S = \int_0^2 \sqrt{1+e^{-2x}}\,dx$$

$$\sqrt{1+e^{-2x}} = \sqrt{e^{-2x}(e^{2x}+1)} = e^{-x}\sqrt{e^{2x}+1}$$

$u = e^x$

$du = e^x dx$

$dx = \frac{du}{e^x} = \frac{du}{u}$

$\frac{1}{u}\sqrt{u^2+1}$

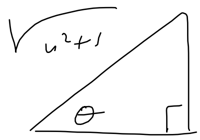
$$\int_{0=x}^{2=x} \frac{\sqrt{u^2+1}}{u} \cdot \frac{du}{u}$$

$u = e^x$

$u = e^2$

$u = e^0 = 1$

$$\int_{1\ (x=0)}^{e^2\ (x=2)} \frac{(u^2+1)^{\frac{1}{2}}\,du}{u^2}$$



$e^x = u = \tan\theta \implies du = \sec^2\theta\,d\theta$

$$\int \frac{\sqrt{\tan^2\theta+1}\ du}{\tan^2\theta} = \int \frac{|\sec\theta|\sec^2\theta\,d\theta}{\tan^2\theta}$$

Not sure if $|\sec\theta| = \pm\sec\theta$. Assume $\sec\theta > 0$ for now. Then drill down.

$$\int \frac{\sec^3\theta\,d\theta}{\tan^2\theta\,d\theta} = \int\left(\frac{1}{\cos^3\theta}\right)\left(\frac{\cos^2\theta}{\sin^2\theta}\right)d\theta$$

$$= \int \frac{d\theta}{\cos\theta\sin^2\theta} = \int \sec\theta\csc^2\theta\,d\theta$$

$-\sin$

$$\int \sec\theta \cot^2\theta \, d\theta + \int \sec\theta \, d\theta$$

$$= \int \cancel{\sec\theta} \frac{\cos^{2}\theta}{\sin^2\theta} \, d\theta + \ln|\sec\theta + \tan\theta|$$

$$= \int \sin^{-2}\theta \cos\theta \, d\theta + \ln \text{———} + C$$

$$= -\sin\theta + \ln|\sec\theta + \tan\theta| + C$$

$u = \sin\theta \quad du = \cos\theta \, d\theta \qquad u = \tan\theta$

$$\int u^{-2} \, du$$

$v = \tan\theta$