$$\int \sqrt{1-\sin x}\, dx$$

$$\frac{\sqrt{(1-\sin x)(1+\sin x)}}{\sqrt{1+\sin x}} = \frac{\sqrt{1-\sin^2 x}}{\sqrt{1+\sin x}} = \frac{\sqrt{\cos^2 x}}{\sqrt{1+\sin x}}$$

$$= \frac{|\cos x|}{\sqrt{1+\sin x}} = \begin{cases} \dfrac{\cos x}{\sqrt{1+\sin x}} & \text{if } \cos x \ge 0 \\ -\dfrac{\cos x}{\sqrt{1+\sin x}} & \text{if } \cos x < 0 \end{cases}$$

$$u = \sqrt{1-\sin x} \qquad dv = dx$$

$$du = \tfrac{1}{2}(1-\sin x)^{-\frac{1}{2}}(-\cos x)\, dx \qquad v = x$$

$$uv - \int v\, du = \left(\sqrt{1-\sin x}\right)(x) + \tfrac{1}{2}\int x \left(\frac{\cos x}{\sqrt{1-\sin x}}\right) dx$$

$$\int \frac{\cos x}{\sqrt{1+\sin x}}\, dx \qquad u = 1+\sin x \qquad du = +\cos x\, dx$$

$$= + \int (1+\sin x)^{-\frac{1}{2}} (+\cos x\, dx)$$

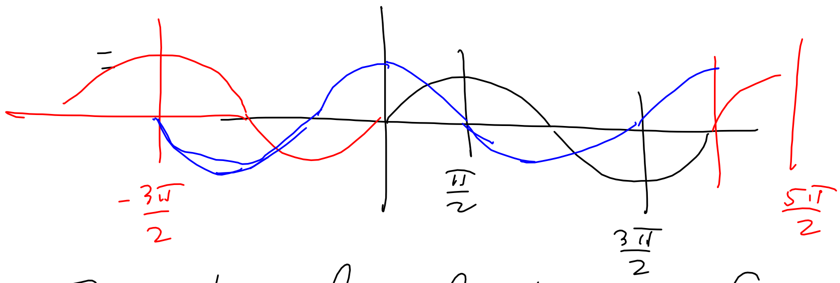
$$= + \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C = 2\sqrt{1+\sin x} + C$$

$$\int \frac{-\cos x}{\sqrt{1+\sin x}}\, dx \qquad u = 1+\sin x \qquad du = \cos x\, dx$$

$$= -\int (1+\sin x)^{-\frac{1}{2}} (\cos x\, dx) = -2\sqrt{\sin x + 1} + C$$

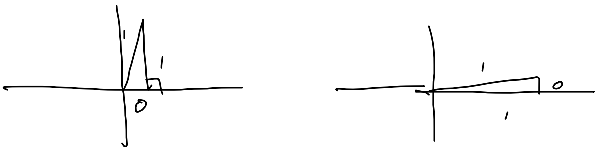
If $-\frac{\pi}{2} < x < \frac{\pi}{2}$

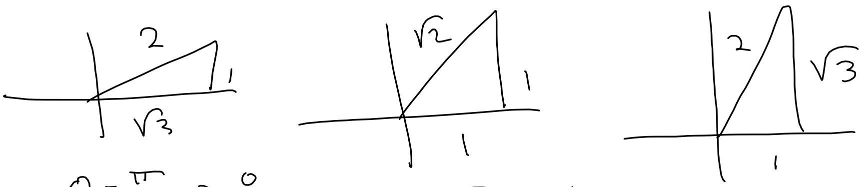
This is legit.



The integral is legit away from $\sin x = 1$

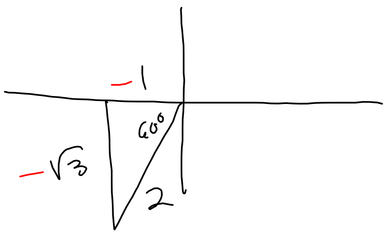
Away from $x = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots, \frac{(4n+1)\pi}{2}$





$\theta = \frac{\pi}{6} = 30^\circ$ $\quad$ $\theta = \frac{\pi}{4} = 45^\circ$ $\quad$ $\theta = \frac{\pi}{3} = 60$

$$\cos\left(\frac{4\pi}{3}\right) = \cos\left(240^\circ\right)$$



$\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

$\cos \frac{4\pi}{3} = -\frac{1}{2}$

$\tan \frac{4\pi}{3} = \sqrt{3}$

FRIDAY TEST:

PART WILL BE TAKE-HOME.

Numerical Integration / Approximate integration

A tougher partial-fractions question involving irreducible quadratic factors

$$\frac{1}{(x^2+2x+3)(x-1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+2x+3}$$

Plan for sit-down is no more than 8 problems. Your strategizing is going to weigh more than final answer.

$$1 = A(x^2+2x+3) + (Bx+C)(x+1)$$

$x=-1$ $\quad A(1^2-2+3) = 2A = 1 \Rightarrow \boxed{A=\frac{1}{2}}$

$x=1$ (Random, small, integer)

$A(6) + \left(B(1)+C\right)(1+1)$

$6A + 2B + 2C = 1$

$3 + 2B + 2C = 1$

$2B + 2C = -2 \Rightarrow \boxed{B + C = -1}$

$x=2:$ $A(2^2+2(2)+3) + \left(B(2)+C\right)(2+1)$

$7A + 6B + 3C = 1$

$\frac{7}{2} + 6B + 3C = 1$

$\boxed{6B + 3C = -\frac{5}{2}}$

$B + C = -1$

$12B + 6C = -5$

$$\left[\begin{array}{cc|c} 1 & 1 & -1 \\ 12 & 6 & -5 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -6 & 7 \end{array}\right]$$

$-6C = 7$

$\boxed{C = -\frac{7}{6}}$

$B + \left(-\frac{7}{6}\right) = -1$

$\boxed{B = \frac{1}{6}}$

So, $$\frac{1}{(x^2+3x+2)(x+1)} = \frac{\frac{1}{2}}{x+1} + \frac{\frac{1}{6}x - \frac{7}{6}}{x^2+2x+3}$$

The main approximate-integral technique will be just do a finite riemann sum and maybe refine it once or twice with a bigger number of rectangles.