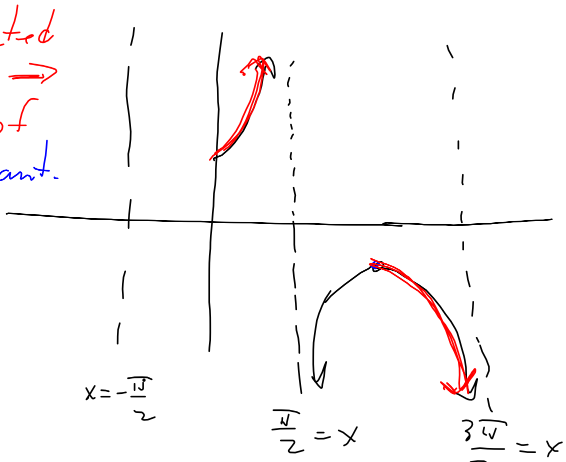
§7.8 #56

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\,\text{arcsec}\left(\frac{u}{a}\right)+C$$

$$\int_2^\infty \frac{dx}{x\sqrt{x^2-4}}$$

Restricted cosine →

Domain of restricted secant.



$$= \int_2^3 + \int_3^\infty$$

$$= \lim_{t\to 2^-} \int_t^3 \frac{dx}{x\sqrt{x^2-4}} + \lim_{t\to\infty} \int_3^t \frac{dx}{x\sqrt{x^2-4}}$$

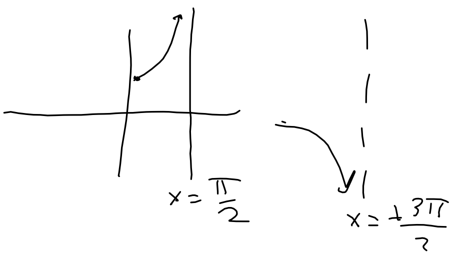
Keenan

$$= \lim_{t\to 2^-} \left[\frac{1}{2}\,\text{arcsec}\left(\frac{x}{2}\right)\right]_t^3 + \lim_{t\to\infty} \left[\frac{1}{2}\,\text{arcsec}(x)\right]_3^t$$

$$t = \frac{1}{2}\,\text{arcsec}\left(\frac{3}{2}\right) - \lim_{t\to 2^-}\left[\frac{1}{2}\,\text{arcsec}\left(\frac{t}{2}\right)\right]$$

$$+ \lim_{t\to\infty} \frac{1}{2}\,\text{arcsec}\left(\frac{t}{2}\right) - \frac{1}{2}\,\text{arcsec}\left(\frac{3}{2}\right)$$

$$= \frac{1}{2}\,\text{arcsec}(1) + \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4}$$



$$\int x^2 \sin x \, dx \qquad u = x^2 \quad du = 2x\,dx \qquad dv = \sin x\,dx \quad v = -\cos x$$

$$= uv - \int v\,du = -x^2\cos x + 2\int x\cos x\,dx \qquad u = x \quad du = dx \qquad dv = \cos x\,dx \quad v = \sin x$$

$$= -x^2\cos x + uv - \int v\,du = -x^2\cos x + x\sin x - \int \sin x\,dx$$

$$= -x^2\cos x + x\sin x + \cos x + C$$

Build a robust cheatsheet $\underline{\underline{\text{OR}}}$ be really good with the Chain Rule

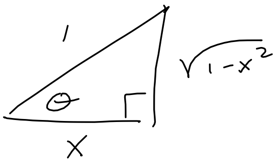
$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{u}{a}\right) + C$$

$$\int \frac{dx}{\sqrt{1-x^2}} \qquad \arcsin(x) + C$$

$$\frac{d}{dx}\left[\arcsin(x)\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\arcsin(u(x))\right] = \frac{u'(x)}{\sqrt{1-u(x)^2}}$$

$$\int \frac{dx}{x\sqrt{1-x^2}}$$



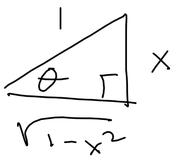
$x = \cos\theta$

$dx = -\sin\theta \, d\theta$

$$\sqrt{1-x^2} = \sqrt{1-\cos^2\theta} = \sqrt{\sin^2\theta} = |\sin\theta|$$

To know if this is $\sin\theta$ or $-\sin\theta$, depending on where θ lives. No specifics?

Make a note about $|\sin\theta|$



$x = \sin\theta$

Assuming $\theta \in QI$:

$$= \int \frac{-\sin\theta \, d\theta}{\cos\theta(\sin\theta)}$$

$$= -\int \sec\theta \, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{1}{x} + \frac{\sqrt{1-x^2}}{x}\right| + C$$

$\ln\left(\frac{A}{B}\right) = \ln(A) - \ln(B)$

$$= \ln|1 + \sqrt{1-x^2}| - \ln|x| + C$$

$$\int \sec\theta \left(\frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}\right) d\theta$$

$$= \int \frac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta} d\theta$$

$$= \int \frac{du}{u}, \text{ where } u = \sec\theta + \tan\theta !$$

$=$

$$\int \sin(2x)\cos(3x)\,dx \qquad \int \sin(mx)\cos(nx)\,dx$$

$$= \int (2\sin x\cos x)(\cos(2x)\cos x - \sin(2x)\sin(x))\,dx$$

$$= 2\int (\sin x)(\cos x)((1-2\sin^2(x))(\cos(x) - 2\sin(x)\cos(x)\sin(x))\,dx$$

$$\cos(2x) = 1 - 2\sin^2 x$$

$$= 2\int \sin x\cos x\,(\cos x - 2\sin^2 x\cos x - 2\sin^3 x\cos x)\,dx$$

$$= 2\int (\sin x\cos^2 x - 2\sin^3 x\cos^2 x - 2\sin^4 x\cos^2 x)\,dx$$

Easy   Easy $u = \cos x$ $du = -\sin x\,dx$

$$= -2\int \cos^2 x\,(-\sin x\,dx) - 4\int (\sin x - \sin x\cos^2 x)\cos^2 x\,dx$$

$$-4\int \sin^4 x\cos^2 x\,dx \quad = -2\frac{\cos^3 x}{3} + 4\frac{\cos^3 x}{3} - 4\frac{\cos^5 x}{5} - 4\int \sin^4 x\cos^2 x\,dx$$

Power-Reduction needed

$$\sin^3 x = \sin x(1-\cos^2 x) = \sin x - \sin x\cos^2 x$$

$$= \frac{2}{3}\cos^3 x - \frac{4}{5}\cos^5 x$$

$$\sin^4 x\cos^2 x = \left(\frac{1-\cos(2x)}{2}\right)^2\left(\frac{1+\cos(2x)}{2}\right)$$

$$= \frac{1}{8}\left[(1 - 2\cos(2x) + \cos^2(2x))(\cos(2x) + 1)\right.$$

$$= \frac{1}{8}\left[\cos(2x) + 1 - 2\cos^2(2x) - 2\cos(2x) + \cos^3(2x) + \cos^2(2x)\right]$$

§7.2

$u = t$ $\quad dv = \sin^2 t \, dt = \frac{1}{2} - \frac{1}{2}\cos(2t)\, dt$

$du = dt$ $\quad v = \frac{1}{2}t - \frac{1}{2}\cdot\frac{1}{2}\sin(2t)$

$\int t \sin^2 t \, dt$ is good. So is $\int t \sin^3 t \, dt$

$u = 2t$
$du = 2dt$
$dt = \frac{1}{2}du$

$= uv - \int v\, du = t\left(\frac{1}{2}t - \frac{1}{4}\sin(2t)\right)$

$- \int \left(\frac{1}{2}t - \frac{1}{4}\sin(2t)\right) dt$

$= \frac{1}{2}t^2 - \frac{1}{4}t\sin(2t) - \frac{1}{4}t^2 + \frac{1}{4}\cdot\frac{1}{2}\left(-\cos(2t)\right) + C$

$= \frac{1}{4}t^2 - \frac{1}{4}t\sin(2t) - \frac{1}{8}\cos(2t) + C$

$= \frac{1}{4}t^2 - \frac{1}{2}t\sin(t)\cos t - \frac{1}{8}\left[2 - 2\sin^2 t\right]$

$$-\frac{t\cos(t)\sin(t)}{2} + \frac{t^2}{4} + \frac{\sin(t)^2}{4}$$

$\cos^2 t = \frac{1 + \cos(2t)}{2}$

$2\cos^2 t = \cos(2t) + 1$

$2\cos^2 t - 1 = \cos(2t)$

$= 2(1 - \sin^2 t) - 1 = 2 - 2\sin^2 t$