$\int e^x \sin x \, dx$ $\quad u = e^x \quad du = e^x dx \quad dv = \sin x \, dx \quad v = -\cos x$

$uv - \int v \, du = e^x(-\cos x) - \int (-\cos x) e^x dx$

$u = e^x \quad du = e^x dx \quad dv = \cos x \, dx \quad v = \sin x$

$= -e^x \cos x + \int e^x \cos x \, dx$

$= -e^x \cos x + \left[uv - \int v \, du\right] = \underline{-e^x \cos x + e^x \sin x} - \underline{\int \sin x \, e^x dx}$

what we started with

$= \int e^x \sin x \, dx$ = original $\Rightarrow$

$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$

$\Rightarrow \int e^x \sin x \, dx = \dfrac{e^x(\sin x - \cos x)}{2}$

Integration

Look for u-sub first.

Approximate & CAS Integration

$$\int_{-5}^{5} e^{-x^2}\,dx$$

$$\frac{b-a}{n} = \frac{5-(-5)}{n} = \frac{10}{n} = \Delta x$$

Right-endpoint:

$$x_k = a + k\Delta x = -5 + k\left(\frac{10}{n}\right) = \frac{-5n+10k}{n}$$

Keenan

$$f(x_k) = e^{-\left(\frac{5n+10k}{n}\right)^2}$$

$$\int_{-5}^{5} e^{-x^2}\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\,\Delta x = \lim_{n\to\infty}\frac{10}{n}\sum_{k=1}^{n} e^{-\left(\frac{-5n+10k}{n}\right)^2}$$

$$e^x = \sum_{k=0}^{\infty}\frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \cdots$$

$$e^{-x^2} = 1 + (-x^2) + \frac{(-x^2)^2}{2} + \frac{(-x^2)^3}{6} + \frac{(-x^2)^4}{24} + \cdots$$

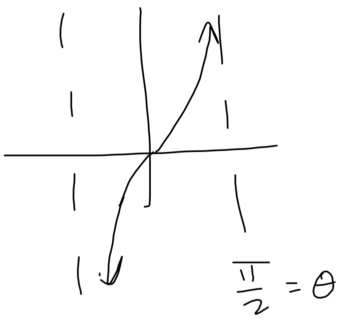
So $$\int_{-5}^{5} e^{-x^2}\,dx \approx \sum_{k=1}^{n}\int_{-5}^{5}\frac{(-x^2)^k}{k!}\,dx$$

$$= \sum_{k=1}^{n}\int_{-5}^{5}\frac{-x^{2k}}{k!}\,dx = \sum_{k=1}^{n}\frac{-x^{2k+1}}{k!(2k+1)}\Bigg]_{-5}^{5}$$

$$\int_0^{\frac{\pi}{2}} \tan^2\theta \, d\theta = \int_0^{\frac{\pi}{2}} (\sec^2\theta - 1)\, d\theta$$

$$\left(\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\cos^2\theta}{\cos^2\theta} = \underset{\text{Duh.}}{\sec^2\theta} - 1 \right)$$

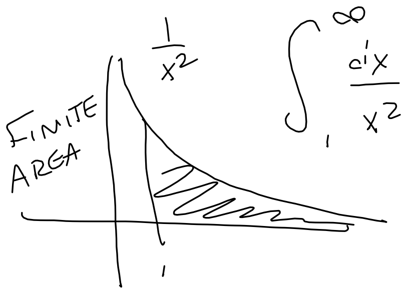
$$= \Big[\tan\theta - \theta \Big]_0^{\frac{\pi}{2}} \quad \nexists$$



p - integrals

$$\int_1^\infty \frac{dx}{x^p} \quad \exists \text{ if } p>1, \quad \nexists \text{ if } p \le 1$$

$$\int_0^1 \frac{dx}{x^p} \quad \exists \text{ if } 0<p<1, \quad \nexists \text{ if } p \ge 1$$



$$\int_1^\infty \frac{dx}{x^2} = \lim_{t\to\infty} \int_1^t \frac{dx}{x^2} = \lim_{t\to\infty} \left[-x^{-1}\right]_1^t$$

$$\int x^{-2}dx = -x^{-1}+C$$

$$= \lim_{t\to\infty}\left[-\frac{1}{x}\right]_1^t = \lim_{t\to\infty}\left(-\frac{1}{t} - \left[-\frac{1}{1}\right]\right) = 1$$

$$\int_0^1 \frac{dx}{x^{1/2}} = \lim_{t\to 0}\int_t^1 x^{-\frac{1}{2}}dx$$

$$= \lim_{t\to 0}\left[2x^{\frac{1}{2}}\right]_t^1 = \lim_{t\to 0}\left(2(1) - 2(t)\right) = 2$$