LEAVE MARGIN

AT TOP LEFT OF

EVERY PAGE.

Leave Room between problems.

DIAGRAMS

GO BIG.

$f(x) = x^2 - 7x - 15$ $\qquad \frac{15}{1} \cdot \frac{4}{4} = \frac{60}{4}$

$= x^2 - 7x + \left(\frac{7}{2}\right)^2 - \frac{49}{4} - \frac{60}{4}$

$= \left(x - \frac{7}{2}\right)^2 - \frac{109}{4} \Rightarrow \mathcal{D}(f) = [\frac{7}{2}, \infty) = \mathcal{R}(f^{-1})$

$\mathcal{R}(f) = [-\frac{109}{4}, \infty)$

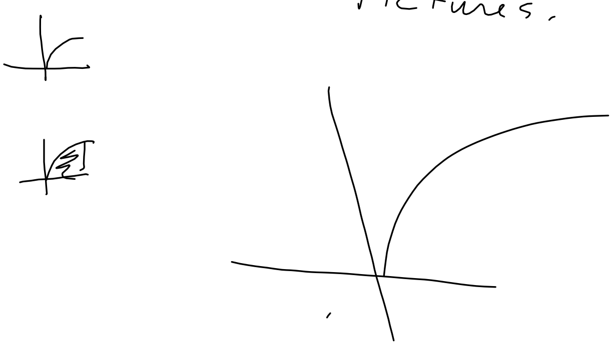
$(\frac{7}{2}, -\frac{109}{4})$

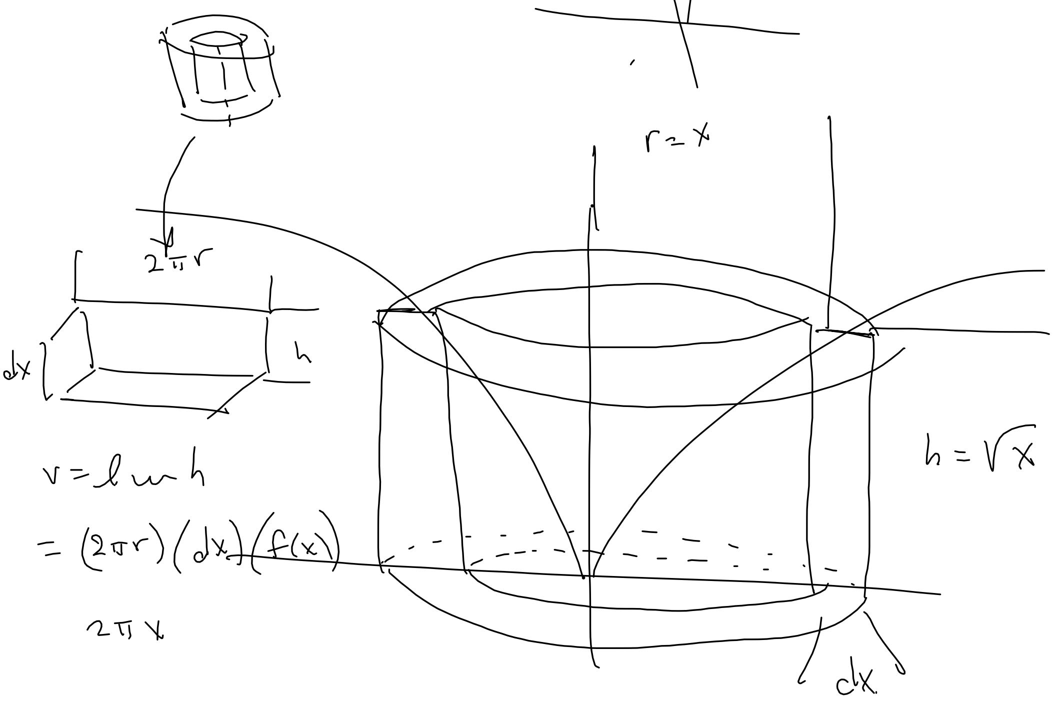
$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}\left[\sin^{-1}(f(x))\right] = \frac{1}{\sqrt{1-(f(x))^2}} \cdot f'(x)$

$\frac{d}{dx}\left[\sin^{-1}(u)\right] = \frac{u'}{\sqrt{1-u^2}}$

BIG COMMENT SMALL OR NONEXISTENT Pictures.



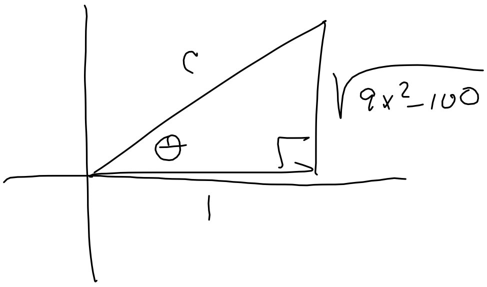


$v = l w h$

$= (2\pi r)(dx)(f(x))$

$2\pi x$

$\sec\left(\tan^{-1}\left(\sqrt{9x^2-100}\right)\right) = \sec\theta$

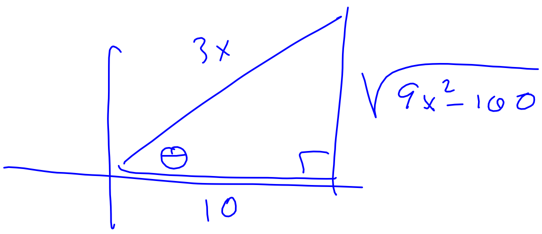


$c^2 = \left(\sqrt{9x^2-100}\right)^2 + 1^2$

$= 9x^2-100+1$

$= 9x^2-99 \Rightarrow c = \sqrt{9x^2-99}$

Then $\sec\theta = \sqrt{9x^2-99}$



$\sec\left(\tan^{-1}\left(\frac{\sqrt{9x^2-100}}{10}\right)\right)$

$\sqrt{9x^2-100}^{\,2} + 10^2$

$= 9x^2-100+100 = 9x^2$

$\Rightarrow c = 3x$

$\boxed{\sqrt{9x^2} = |3x|}$

but for these, just assume everything's positive.

FACT

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} \qquad u=\frac{1}{x}$$

$$= \lim_{u\to 0}(1+u)^{\frac{1}{u}}$$

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{5x} = \lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{5\cdot 3\cdot\frac{x}{3}}$$

$$= \lim_{x\to\infty}\left(\left(1+\frac{3}{x}\right)^{\frac{x}{3}}\right)^{15} = e^{15}\ !$$

$$\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^{5x} = y \Rightarrow$$

$$\ln(y) = \ln\left(\lim_{x\to\infty}\left(\left(1+\frac{3}{x}\right)^{5x}\right)\right) \qquad 2b = \frac{b}{\left(\frac{1}{2}\right)}$$

$$= \lim_{x\to\infty}\left(\ln\left(\left(1+\frac{3}{x}\right)^{5x}\right)\right)$$

$$= \lim_{x\to\infty}\left(5x\ln\left(1+\frac{3}{x}\right)\right) = \infty\cdot 0$$

$$= \lim_{x\to\infty}\left(\frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{5x}}\right) = \frac{0}{0}$$

$$\overset{L'H}{=} \lim_{x\to\infty}\left(\frac{\left(\frac{-\frac{3}{x^2}}{1+\frac{3}{x}}\right)}{-\frac{1}{5x^2}}\right) \qquad \frac{\frac{0}{1}}{0} = \frac{0}{0}$$

$$= \lim_{x\to\infty}\left(\frac{\frac{3}{\cancel{x^2}}}{\frac{1}{5\cancel{x^2}}\left(1+\frac{3}{x}\right)}\right) = 15$$

$$\ln(y) = 15 \Rightarrow \boxed{y = e^{15}}$$

$$\int_1^4 \sqrt{y}\, \ln(y)\, dy$$

$u = \ln y \qquad dv = y^{\frac{1}{2}} dy$

$du = \frac{1}{y} dy \qquad v = \frac{2}{3} y^{\frac{3}{2}}$

$$\int_a^b u\, dv = uv\Big|_a^b - \int_a^b v\, du$$

$$\frac{2}{3} y^{\frac{3}{2}} \ln y \Big]_1^4 - \int_1^4 \frac{2}{3} y^{\frac{3}{2}} \cdot y^{-1} dy$$

$$= \frac{2}{3}\left[4^{\frac{3}{2}} \ln 4 - 1^{\frac{3}{2}} \ln 1\right] - \frac{2}{3}\int_1^4 y^{\frac{1}{2}} dy$$

$$= \frac{2}{3}\left[8 \ln 4\right] - \frac{2}{3}\left[\frac{2}{3} y^{\frac{3}{2}}\right]_1^4 = \frac{16}{3}\ln 4 - \frac{4}{9}\left[8 - 1\right]$$

$$= \boxed{\frac{16}{3}\ln 4 - \frac{28}{9}}$$

$$A = \int \frac{dx}{5x\sqrt{x^2-36}} = \frac{1}{5}\cdot\frac{1}{6}$$

u^2-1

$$\int \frac{du}{u\sqrt{u^2-a^2}}$$

$$\sqrt{x^2-36} = \sqrt{36\left(\frac{x^2}{36}-1\right)}$$

$$= 6\sqrt{\left(\frac{x}{6}\right)^2-1} \Rightarrow$$

$$A = \frac{1}{30}\int \frac{dx}{x\sqrt{\left(\frac{x}{6}\right)^2-1}}$$

Let $u = \frac{x}{6} \Rightarrow x = 6u$

$du = \frac{1}{6}\,dx \Rightarrow dx = 6\,du$

$$= \frac{1}{30}\int \frac{6\,du}{6u\sqrt{u^2-1}} = \frac{1}{30}\int \frac{du}{u\sqrt{u^2-1}}$$

$$= \frac{1}{30}\sec^{-1}(u) + C = \frac{1}{30}\sec^{-1}\left(\frac{x}{6}\right) + C$$

Heads-up on Theory for §7.7
I'm pretty sure I got the function
and the quadratic approximations mixed-up
in my talk. (Simpson's Rule)

Test 1 Panickers:
Get a re-take, last week
of school.

$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left[\sin^{-1}(u)\right] = \frac{u'}{\sqrt{1-u^2}}$$

SAME

$$\frac{d}{dx}\left[\sin^{-1}(g(x))\right] = \underbrace{\frac{1}{\sqrt{1-(g(x))^2}}}_{\frac{df}{dg}} \cdot \underbrace{g'(x)}_{\frac{dg}{dx}}$$

OR $f'(g) \cdot g'(x)$

Section 7.5, 7.6 Due Wednesday.

Section 7.7 Due Friday

Section 7.8 finishing-up Next Monday

Test 2 Next Wednesday OR

Test 2 Next Friday AND turn in 8.1 on Monday