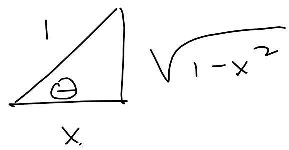
§7.1 #7 notes messed-up. Video is "fixed"

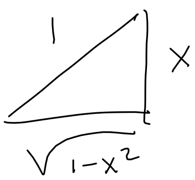
$\int e^x \sin x \, dx$ Integrate by parts twice.
Do algebra.

§7.2 Trig Integrals Ask.

$\sqrt{1-x^2}$

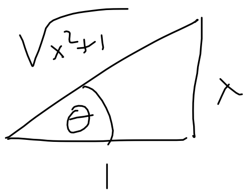


$x = \cos\theta$

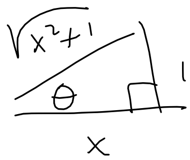


$x = \sin\theta$

$\sqrt{x^2+1}$



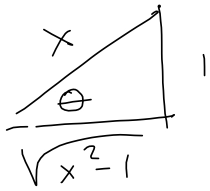
$x = \tan\theta$
Sweet.



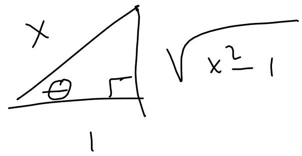
$x = \cot\theta$ Meh

Preferred subs are

$\sqrt{x^2-1}$

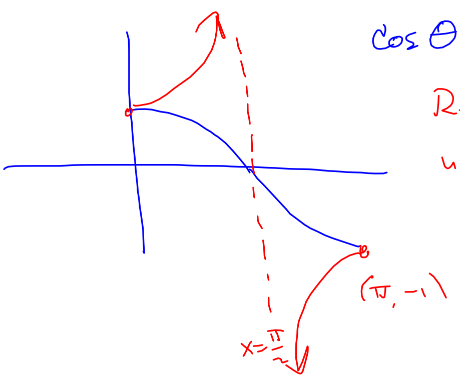


$x = \csc\theta$



$x = \sec\theta$

Not much examination of pathology, like Domain issues. May be one or two exercises. But what if it's a DEFINITE INTEGRAL that's defined, but your substitution creates a domain for θ where $\sec\theta$ blows up.



$\cos\theta$

Restricted $\cos\theta$ gives us restricted

$x = \sec\theta$ $\quad 0 \le \theta < \frac{\pi}{2}$

$\frac{\pi}{2} < \theta \le \pi$

Get domains, AS NEEDED, by knowing the pictures & practicing them.

Learn to engage "lightly" w/ new material.

You don't have to master it on a first pass, but make that first pass.

$\sqrt{1-x^2}$

$\sqrt{9-x^2} = \sqrt{9 - 9\left(\frac{x^2}{9}\right)}$

$= 3\sqrt{1-\left(\frac{x}{3}\right)^2} \qquad u = \frac{x}{3}$

$3\sqrt{1-u^2}$ BACK TO HERE

$\sqrt{ax^2+bx+c}$

Ugh $\sqrt{x^2+4x-7} = \sqrt{x^2+4x+2^2-4-7}$

Better $= \sqrt{(x+2)^2-11} \qquad u = x+2$

Warmer $= \sqrt{u^2-11}$

$= \sqrt{11\left(\frac{u^2}{11}\right)-11}$

$= \sqrt{11}\sqrt{\left(\frac{u}{\sqrt{11}}\right)^2-1}$ Meh

$= \sqrt{11}\sqrt{v^2-1} \qquad v = \frac{u}{\sqrt{11}}$

Doable

Polynomial

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_2 x^2 + a_1 x + a_0$$

$$= a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1} + a_n x^n$$

$$= \sum_{k=0}^{n} a_k x^k = p(x)$$

Rational Function

$$\frac{p(x)}{q(x)} = \frac{\sum_{k=0}^{n} a_k x^k}{\sum_{k=0}^{m} b_k x^k}$$

1. $n \geq m$: Improper. Divide
2. $m > n$ Proper. Factor Denominator & do partial fractions

$$\frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$ Find A, B.

$$\frac{1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{Bx+C}{(x+1)^2} + \frac{C}{x+2}$$ Find A, B, C, D

$x^2+3x+2 = (x+2)(x+1)$

$a=1, b=3, c=2$

$b^2-4ac = 3^2-4(1)(2) = 9-8=1$

$x = \frac{-3\pm\sqrt{1}}{2} = \frac{-3\pm 1}{2}$ $\to$ $\frac{-2}{2} = -1$; $\frac{-4}{2} = -2$

$\Rightarrow f(x) = (x+1)(x+2)$

$x^2 - 11x + 7$

$x^2 - 11x + \left(\frac{11}{2}\right)^2 - \frac{121}{4} + \frac{28}{4}$

$= \left(x-\frac{11}{2}\right)^2 - \frac{93}{4} = 0 \Rightarrow$

$x - \frac{11}{2} = \pm\frac{\sqrt{93}}{2}$

$3\,\overline{)93}$
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$x = \frac{11\pm\sqrt{93}}{2}$

$\left(x-\left(\frac{11+\sqrt{93}}{2}\right)\right)\left(x-\left(\frac{11-\sqrt{93}}{2}\right)\right)$

FACTOR THEOREM

MOAR 7.4

$f(x) = \frac{1}{x^2-5x+6}$ Evaluate $\int f(x)\,dx$

$f(x) = \frac{1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$

THE BASIC SKILL

$\Longrightarrow 1 = A(x-3) + B(x-2)$

$x=3$: $\boxed{1 = B}$

$\int_0^\infty e^{-x^2}\,dx$

$x=2$ $1 = A(-1) \Longrightarrow A = -1$,

Decomposition of $f(x)$

so $f(x) = \frac{A}{x-2} + \frac{B}{x-3}$

$= \frac{-1}{x-2} + \frac{1}{x-3}$ ∎

$\int f(x)\,dx = -\ln|x-2| + \ln|x-3| + C$

You might glance at:

Cauchy Integral Theorem &

Calculus of residues

we'd say $x = 2, 3$ are "poles" &

poles are where there's a residue.

$$e^x = \sum_{k=0}^{\infty} \frac{x^n}{n!} = x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots$$