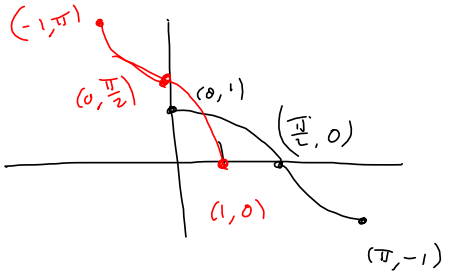
Restricted Cosine

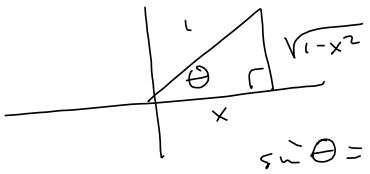



$\cos^{-1}(x)$

$\cos(x)$

$(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}$

$\dfrac{d}{dx}\left[\cos^{-1}(x)\right] = \dfrac{1}{-\sin(\cos^{-1}(x))}$




$\sin\theta = \dfrac{\sqrt{1-x^2}}{1}$

$= \dfrac{1}{-\sin\theta}$

$= -\dfrac{1}{\sqrt{1-x^2}}$

**11 Table of Derivatives of Inverse Trigonometric Functions**

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

$\int \frac{dx}{x\sqrt{x^2-4}}$ This would depend on domain; otherwise, we have 2 things from which to choose:

$\int -\frac{dx}{x\sqrt{x^2-1}} = \csc^{-1}x + C$

$\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1}x + C$

Need to manipulate this to fit:

$x^2-4 = 4\left(\frac{x^2}{4}-1\right)$

$= 4\left(\left(\frac{x}{2}\right)^2-1\right)$

$= 4(u^2-1)$

$u = \frac{x}{2} \Rightarrow x = 2u$, so

My solutions were off by a factor of 4.

$2\int \ldots$ no. $\frac{1}{2}\int \ldots$ yes.

$$\int \frac{dx}{x\sqrt{x^2-4}} = \int \frac{dx}{2u\sqrt{4(u^2-1)}} \quad \cdot \; x = 2u \Rightarrow dx = 2\,du$$

$$= \int \frac{2\,du}{2u\cdot 2\sqrt{u^2-1}} = \frac{1}{2}\int \frac{du}{u\sqrt{u^2-1}} = \frac{1}{2}\sec^{-1}(u) + C$$

$$= \frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right) + C$$

trig funcs in terms of $e^x$:

$\sinh x = \frac{e^x - e^{-x}}{2}$. But did you know $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$?

$\int \arctan\left(\frac{1}{x}\right) dx$ $\qquad \frac{d}{dx}[\arctan x] = \frac{1}{x^2+1}$

$\int \tan^{-1}(x)\, dx$ $\qquad u = \arctan\left(\frac{1}{x}\right)$

$du = \left(\frac{1}{\left(\frac{1}{x}\right)^2+1}\right)\left(-\frac{1}{x^2}\right) dx = -\frac{1}{x^2+1} dx$

$\int u\, dv = uv - \int v\, du$ $\qquad dv = dx$

$v = x$

$= x\arctan\left(\frac{1}{x}\right) - \int x\left(\frac{1}{x^2+1}\right) dx$

$u = x^2+1$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$= x\arctan\left(\frac{1}{x}\right) - \frac{1}{2}\int\left(\frac{1}{x^2+1}\right)(2x)\, dx$

$= x\arctan\left(\frac{1}{x}\right) - \frac{1}{2}\int \frac{du}{u}$ $= x\arctan\left(\frac{1}{x}\right) - \frac{1}{2}\ln(x^2+1) + C$