Old Test 1 stuff

$f(x) = x^2+5x+11$

$= x^2+5x+\left(\frac{5}{2}\right)^2 - \frac{25}{4} + \frac{44}{4}$

$= \left(x+\frac{5}{2}\right)^2 + \frac{19}{4}$ $(h,k) = \left(-\frac{5}{2}, \frac{19}{4}\right)$

Restrict $\mathcal{D}(f) = \left[-\frac{5}{2}, \infty\right)$ to make it 1-to-1.

(a) Find f^{-1}

$\left(x+\frac{5}{2}\right)^2 + \frac{19}{4} = y$

$\left(x+\frac{5}{2}\right)^2 = y - \frac{9}{4}$ Good eye Isaac!

$x+\frac{5}{2} = \pm\sqrt{y-\frac{9}{4}} \longrightarrow \sqrt{y-\frac{9}{4}}$

$x = \sqrt{y-\frac{9}{4}} - \frac{5}{2} \Longrightarrow \boxed{f^{-1}(x) = \sqrt{x-\frac{9}{4}} - \frac{5}{2}}$

(b) Find $\left(f^{-1}(x)\right)' = (f^{-1})'(x)$

$= \boxed{\frac{1}{2}\left(x-\frac{9}{4}\right)^{-\frac{1}{2}}}$

(c) $(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$ Instead, find $(f^{-1})'(5)$

$f(x) = x^2+5x+11 \Longrightarrow f'(x) = 2x+5$

$(f^{-1})'(x) = \frac{1}{2f^{-1}(x)+5}$

Pretend you never found $f^{-1}(x)$, directly

You CAN find $(f^{-1})'(5)$, by finding $f^{-1}(5)$! How to find that?

Set $f(x)=5$! Solve!

$x^2+5x+11 = 5$ Want $f^{-1}(5)$!

$x^2+5x+6 = 0$ $x \in \{\text{some stuff}\}$

$(x+3)(x+2) = 0$ cache

$x \in \{-3,-2\}$

$-3 \notin$ restricted $\mathcal{D}$, so

$x = -2$!

So, $(f^{-1})'(5) = \frac{1}{2(-2)+5} = \frac{1}{1} = 1$!

Check:

$(f^{-1})'(5) = \frac{1}{2}\left(x-\frac{9}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{8-9}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}$

$= \frac{1}{2}(4)^{\frac{1}{2}} = \frac{1}{2}(2) = 1$

Isaac & the whole 3rd row hate me.

This is fishy

$f'(f^{-1}(x))$ ✓

But doing it directly failed?

$(f^{-1})'(5)$ $f^{-1}(x) = \sqrt{x-\frac{19}{4}} - \frac{5}{2}$

So, $(f^{-1})'(x) = \frac{1}{2}\left(x-\frac{19}{4}\right)^{-\frac{1}{2}}$

& $(f^{-1})'(5) = \frac{1}{2}\left(\frac{20-19}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{4}\right)^{-\frac{1}{2}}$

$= \frac{1}{2}(4)^{\frac{1}{2}} = \frac{1}{2}(2) = 1$, See? I tolt ya, Charlie.

b. $y = \ln\left(\dfrac{\sqrt[5]{x^2-3x}}{(3x^5+5x)^3}\right) = \frac{1}{5}\ln(x^2-3x) - 3\ln(3x^2+5x) \Rightarrow$

Miscopy!

$\to 5$

$$\boxed{y' = \frac{1}{5}\left(\frac{2x-3}{x^2-3x}\right) - 3\left(\frac{6x+5}{3x^2+5x}\right)}$$

$\to 5$

$15x^4+5$

x^2-3x
$= x(x-3)$
$3x^2+5x = x(3x+5)$

$$y = \frac{1}{5}\left[\ln x + \ln(x-3)\right] - 3\left[\ln x + \ln(3x+5)\right]$$

$$\Rightarrow y' = \frac{1}{5}\left[\frac{1}{x} + \frac{1}{x-3}\right] - 3\left[\frac{1}{x} + \frac{3}{3x+5}\right]$$

$$a^x a^y = a^{x+y} \longleftrightarrow \ln(xy) = \ln x + \ln y$$

$$(a^b)^c = a^{bc} \longleftrightarrow \ln(b^c) = c\ln b$$

so $\dfrac{a^x}{a^y} = a^x a^{-y} = a^{x-y}$

$$\ln\left(\frac{b}{c}\right) = \ln(bc^{-1}) = \ln b + \ln(c^{-1})$$
$$= \ln b - \ln c$$

Reason out $\frac{d}{dx}\left[\cos^{-1}(x)\right]$

$f^{-1}(x) = \cos^{-1}(x)$ so

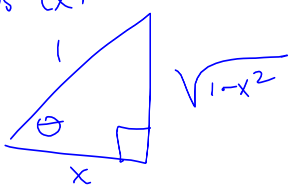
$f(x) = \cos x$, amirite?

$f'(x) = -\sin x$

So $(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))} = \dfrac{1}{-\sin(\cos^{-1}(x))}$

The angle whose cosine is x.

$\theta = \cos^{-1}(x)$



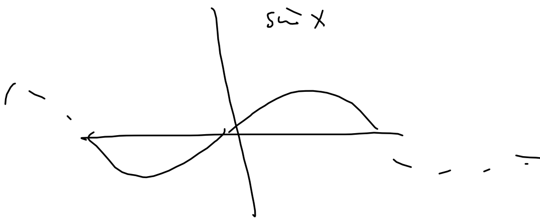
$= \dfrac{1}{-\sqrt{1-x^2}}$

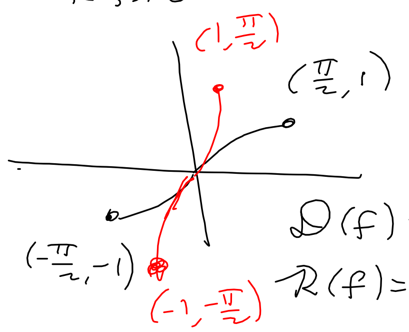
Inverse trig funcs.

Never Memorize Domains & Ranges
work them out from the restrictions
we place on f(x) to make it 1-to-1.

Restriction

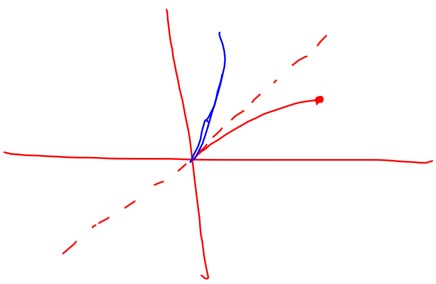
$\sin^{-1}(x)$

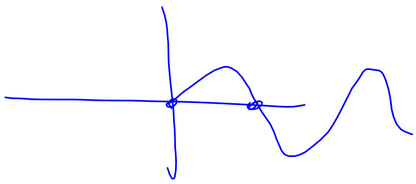


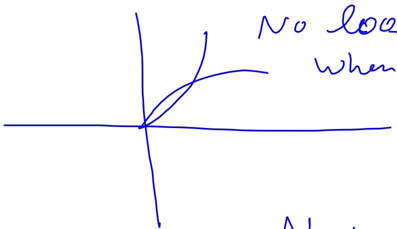


$$\mathcal{D}(f) = \mathcal{R}(f^{-1}) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

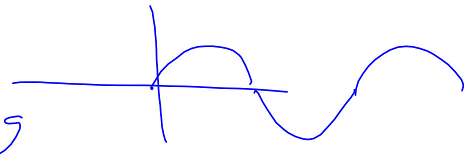
$$\mathcal{R}(f) = \mathcal{D}(f^{-1}) = [-1, 1]$$



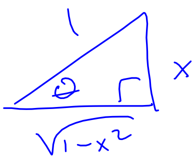




No loop when graphing $\sin x$ & $\sin^{-1}(x)$



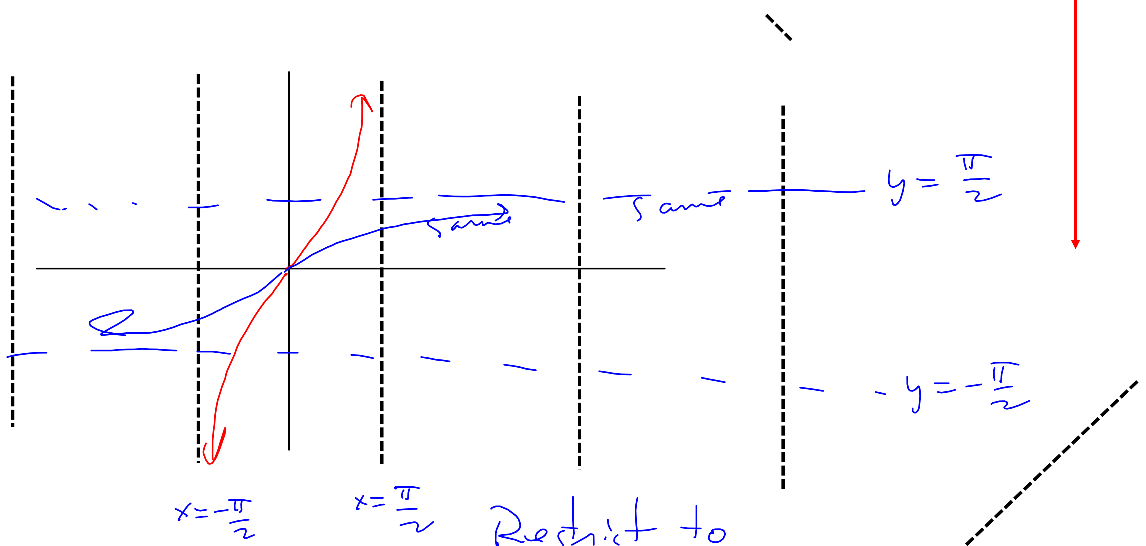
$\theta = \sin^{-1}(x)$



Now, $$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\cos\left(\sin^{-1}(x)\right)} = \frac{1}{\cos\theta}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

$f(x) = \tan x$

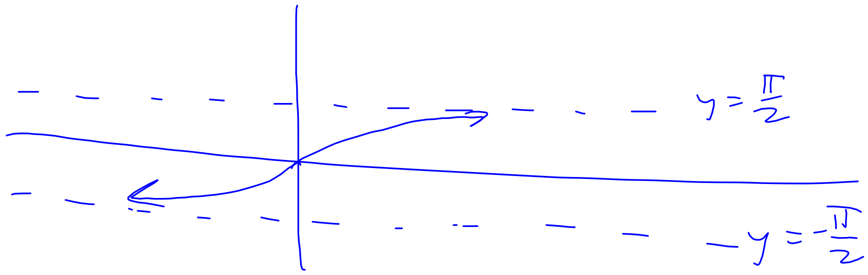


Restrict to

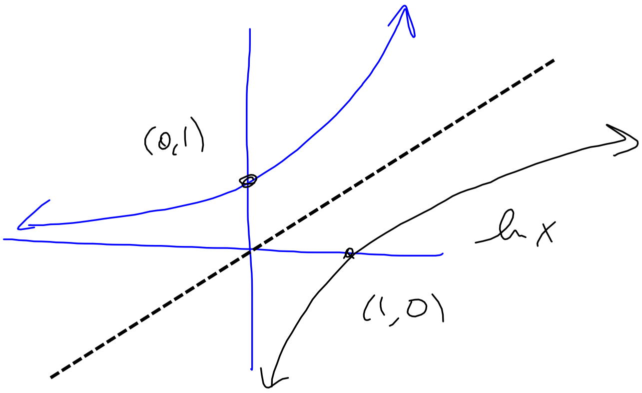
$\mathcal{D}(\tan x) = \mathcal{R}(\tan^{-1} x = \arctan x) = (-\frac{\pi}{2}, \frac{\pi}{2})$

$\mathcal{R}(\tan x) = \mathcal{D}(\tan^{-1} x) = (-\infty, \infty)$

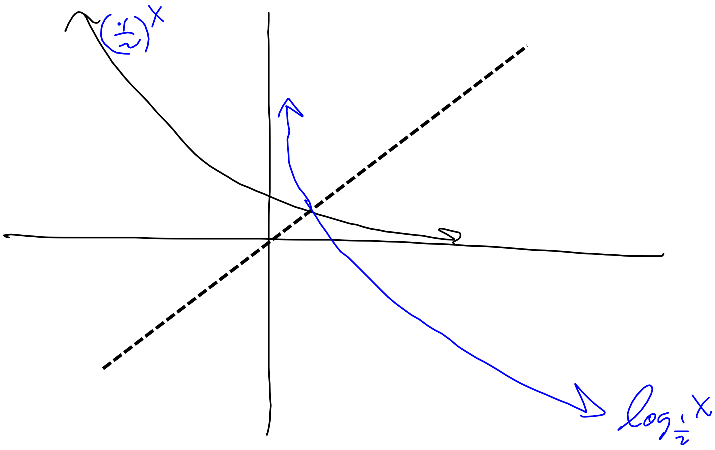
Sketch of $\tan^{-1}(x)$



e^x & $\ln x$



$\left(\frac{1}{2}\right)^x$ & $\log_{\frac{1}{2}}(x)$



L'Hôpital

$$\lim_{x\to 0} \frac{e^x-1-x}{x^2} = \frac{0}{0} ?!$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{e^x-1}{2x} = \frac{0}{0} ?!$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{e^x}{2} = \frac{e^0}{2} = \frac{1}{2}$$

Logarithmic Differentiation: Find $\frac{dy}{dx}$

$$y = \frac{(2x^2-5)^{2/3}(x^3+7)^5}{\sqrt[3]{(3x-\sin x)^{11}}} \Longrightarrow$$

$$\frac{d}{dx}\left[\ln y = \frac{2}{3}\ln(2x^2-5) + 5\ln(x^3+7) - \frac{11}{3}\ln(3x-\sin x)\right] \Longrightarrow$$

$$\frac{y'}{y} = \frac{2}{3}\left(\frac{4x}{2x^2-5}\right) + 5\left(\frac{3x^2}{x^3+7}\right) - \frac{11}{3}\left(\frac{3-\cos x}{3x-\sin x}\right) \Longrightarrow$$

$$y' = \left[\frac{2}{3}\left(\frac{4x}{3x^2-5}\right) + \frac{5x^2}{x^3+7} - \frac{11}{3}\left(\frac{3-\cos x}{3x-\sin x}\right)\right] \cdot \left(\frac{(2x^2-5)^{2/3}(x^3+7)^5}{(3x-\sin x)^{11/3}}\right)$$

WRITE MUCH

THINK LITTLE

L'Hopital for ∞^0 case

$$y = x^{\frac{1}{x}} \xrightarrow{x \to \infty} \infty^0 \; ?!$$

$$y = x^{\frac{1}{x}} \Rightarrow \ln y = \ln\left(x^{\frac{1}{x}}\right) = \frac{1}{x} \ln x \xrightarrow{x \to \infty} 0 \cdot \infty$$

$$= \frac{1}{x} \ln x = \frac{\ln x}{x} \xrightarrow{x \to \infty} \frac{\infty}{\infty}$$

$$\underset{x \to \infty}{\xrightarrow{L'H}} \frac{\frac{1}{x}}{1} \xrightarrow{x \to \infty} 0.$$

This says

$$\lim_{x \to \infty} \ln y = \lim_{x \to \infty} \ln\left(x^{\frac{1}{x}}\right) = 0, \text{ so}$$

FACT:

$$\lim_{x \to \infty} y = e^0 = \boxed{1 = \lim_{x \to \infty} y}$$

$\ln x$ is cnt^s,

so $\lim(\ln(x)) = \ln(\lim x)$

$$\left(\begin{array}{c} \ln y = 0 \\ e^{\ln y} = e^0 \\ y = 1 \end{array} \right)$$

If $f(x)$ is cnt^s, then

$$\lim_{x \to c} [f(x)] = f\left(\lim_{x \to c} x\right)$$

$$f(u) = \frac{\ln u}{\ln(2u)+1}$$

$$\frac{d}{du}\left[\ln(2u)\right] = \frac{2}{2u}$$

$$f'(u) = \frac{\frac{1}{u}(\ln(2u)+1) - (\ln u)(\frac{1}{u})}{(\ln(2u)+1)^2}$$

$$= \frac{\frac{1}{u}\left[\ln(2u)+1-\ln(u)\right]}{(\sim)^2} = \frac{\frac{1}{u}\left[\boxed{\ln 2+\ln u}+1-\ln u\right]}{(\ln(2u)+1)^2}$$

$$= \frac{\ln(u)+1+\ln(2)}{u(\ln(2u)+1)^2}$$

$$\ln(xy) = \ln x + \ln y$$

$$a^x a^y = a^{x+y}$$