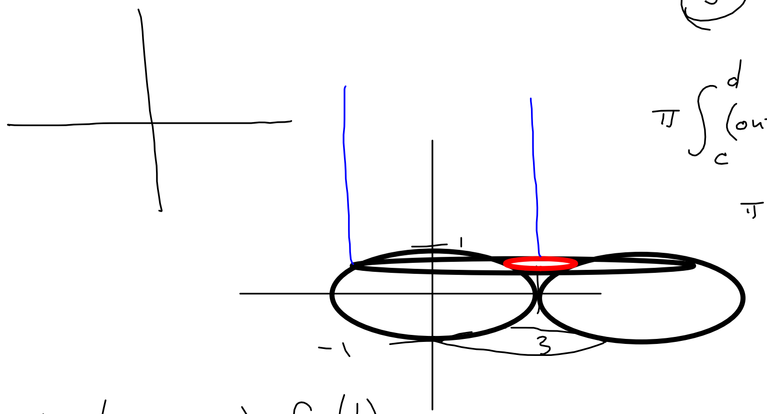
$\boxed{\text{5.2 \#12 b}}$

$x^2 + 9y^2 = 9$

$\frac{x^2}{9} + y^2 = 1$

Revolve about
(a) the line $y = 3$
(b) the line $x = 3$



$\pi \int_c^d (\text{outer}^2 - \text{inner}^2)\, dy$

$\pi \int_{-1}^{1}$

Need $x = g(y)$ for (b)

$\frac{x^2}{9} = 1 - y^2 \Rightarrow x^2 = 9\sqrt{1-y^2} \Rightarrow x = \pm 3\sqrt{1-y^2}$

$\pi \int_{-1}^{1} \left(\left(3 - (-3\sqrt{1-y^2})\right)^2 - \left(3 - 3\sqrt{1-y^2}\right)^2 \right) dy$

$= 2\pi \int_0^1 \left(9 + 18\sqrt{1-y^2} + 9(1-y^2) - \left[9 - 18\sqrt{1-y^2} + 9(1-y^2) \right] \right) dy$

$= 2\pi \int_0^1 36\sqrt{1-y^2}\, dy = 72\pi \int_0^1 \sqrt{1-y^2}\, dy$

$= 72\pi \left[\frac{1}{4}\pi \right] = 18\pi^2$

$x^2 + y^2 = 1$

$x = \pm\sqrt{1-y^2}$

$+\sqrt{1-y^2}$ RIGHT $\frac{1}{2}$

$-\sqrt{1-y^2}$ left $\frac{1}{2}$

$\int_0^1 \sqrt{1-y^2}\, dy = \frac{1}{4}$ - Area of circle of radius 1.

$\frac{1}{4}\left[\pi (1)^2\right]$

§6.5 ish Derivative of the inverse

$f(x) \longleftrightarrow f^{-1}(x)$

Bijections

1-to-1 & onto.

$f: A \xrightarrow[\text{1-to-1}]{\text{ONTO}} B$ $\qquad f^{-1}: B \xrightarrow[\text{1-to-1}]{\text{ONTO}} A$

We want to discuss $\frac{d}{dx}\left[f^{-1}(x)\right]$

$y = f(x) \longrightarrow$

$f^{-1}(y) = x \longrightarrow$

See next page

$\frac{d}{dx}\left[f^{-1}(y)\right] = \frac{d}{dx}[x] = 1$

$= \frac{d}{dy}\left[f^{-1}(y)\right]\frac{dy}{dx} = 1 \longrightarrow$ Missing a key step.

$\frac{dy}{dx} = \frac{1}{\frac{d}{dy}\left[f^{-1}(y)\right]} = \frac{1}{\frac{d}{dy}[x]}$

$$\boxed{\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}}$$

THE formula

Punch line.

$\frac{d}{dy}\left[f^{-1}(y)\right] = f'(f^{-1}(x))$

Test 1: C5 stuff in the Bonus

C6 stuff "regular"

Seeing some good stuff on homework.

Spread things out. write bigger.

" dark.

$$f(f^{-1}(x)) = x$$

$$\frac{d}{dx}\left[f(f^{-1}(x))\right] = 1$$

here's the (a) proper derivation

$$\frac{df}{d(f^{-1}(x))} \cdot \frac{d(f^{-1}(x))}{dx} = 1$$

$$\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{\frac{df}{d[f^{-1}(x)]}} = \frac{1}{f'(f^{-1}(x))}$$

$$f(x) = \sin x = y$$

$$x = \sin^{-1}(y)$$

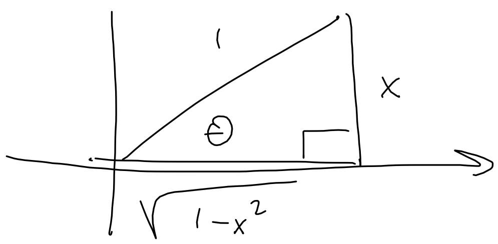
$$f(x) = \sin x$$

$$f'(x) = \cos x$$

$$\frac{d}{dx}\left[\sin^{-1}(x)\right] = \frac{1}{\cos(\sin^{-1}(x))}$$

$$f'(f^{-1}(x)) = \cos\left(\sin^{-1}(x)\right)$$

$$\cos\left(\sin^{-1} x\right) = \cos\theta$$



$$\cos\theta = \sqrt{1-x^2}$$

so, $\frac{d}{dx}\left[\sin^{-1}(x)\right]$

$$= \frac{1}{\cos(\sin^{-1}(x))} = \frac{1}{\sqrt{1-x^2}} \; !$$

14. A thick cable, 60 ft long and weighing 180 lb, hangs from a winch on a crane. Compute in two different ways the work done if the winch winds up 25 ft of the cable.

(a) Follow the method of Example 4.

(b) Write a function for the weight of the remaining cable after x feet has been wound up by the winch. Estimate the amount of work done when the winch pulls up Δx ft of cable.

Let x = # of feet of chain we've lifted.

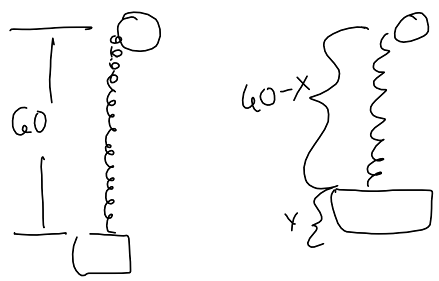
60ft, 180 lbs $\longrightarrow$ $\frac{180 \text{ lbs}}{60 \text{ ft}} = 3 \frac{\text{lbs}}{\text{ft}}$ = linear density

weight of cable lifted is $w(x) = 3x$

(b) weight of remaining cable is $180 - 3x$

WORK = $W = F \cdot D =$

(a)



weight $= (60-x)(3)$

From x to $x+\Delta x$

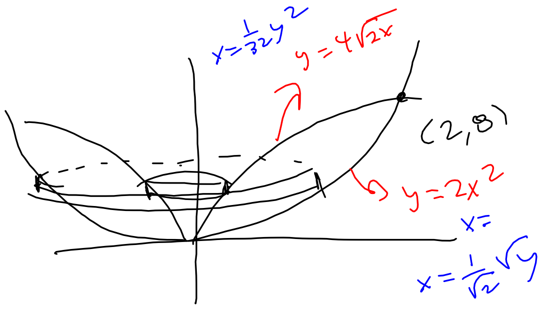
we have $\boxed{3(60-x)\Delta x}$

$3\Delta x \sum_{k=1}^{n} (60-x)$

$3(60-x)\Delta x$ is work done lifting it Δx distance

$\xrightarrow{n \to \infty} 3\int_0^{60} (60-x)\Delta x$

$f(x) = 4\sqrt{2x}$ & $g(x) = 2x^2$ about y-axis



$4\sqrt{2x} = 2x^2$

$16(2x) = 4x^4$

$4x^4 - 32x = 0$

$4x[x^3 - 8] = 4x$

$= 4x[x-2](x^2+2x+4)$

$x = 2$

$\pi \int_0^8 (\text{outer}^2 - \text{inner}^2)\, dy$

$y = 4\sqrt{2x}$

$y^2 = 16 \cdot 2x$

$x = \frac{1}{32}y^2$

$y = 2x^2$

$2x^2 = y$

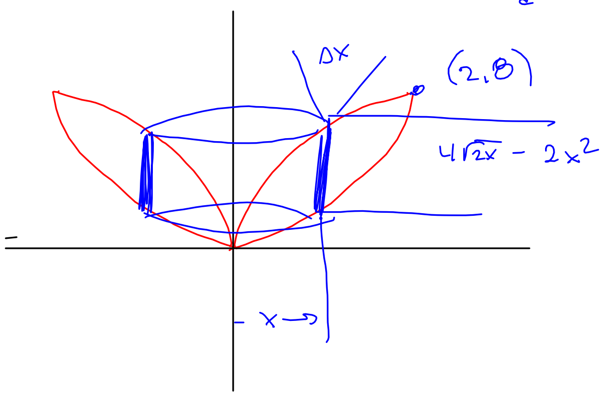
$x^2 = \frac{1}{2}y$

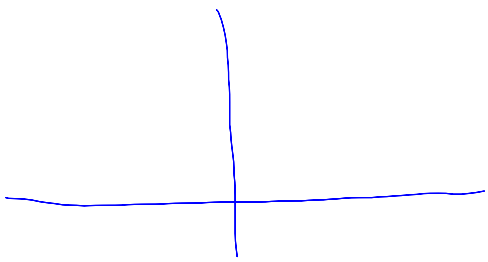
$x = \pm \frac{1}{\sqrt{2}}\sqrt{y}$ → want "+"

$\sqrt{(y/2)}$

$V = \pi \int_0^8 \left(\left(\frac{1}{\sqrt{2}}\sqrt{y}\right)^2 - \left(\frac{1}{32}y^2\right)^2 \right) dy$ Disk Method

Shell method $2\pi \int_a^b x\, y\, dx$ about y-axis





$2\pi \int_0^2 x\left[4\sqrt{2}\, x^{\frac{1}{2}} - 2x^2\right] dx$