§6.1 # 28

Some folks did this one by quadratic formula, which works, but you really want to make completing the square a quick-twitch sort of thing:

$f(x) = y = 2x^2 - 8x$

$= 2(x^2 - 4x \quad)$

$= 2(x^2 - 4x + 2^2) - 2(2)^2$ SWIDT ?

$= 2(x-2)^2 - 8 \longrightarrow$ Vertex $= (h, k) = (2, -8)$

✓ f^{-1} ↘ Solve $= 0$ So $y \geq -8$

$= 0 \Rightarrow$

$= 2(x-2)^2 - 8 = y$

$2(x-2)^2 = y + 8$

$(x-2)^2 = \frac{y+8}{2}$

$x - 2 = \pm\sqrt{\frac{y+8}{2}}$

$x = 2 \pm \sqrt{\frac{y+8}{2}}$

$2(x-2)^2 = 8$

$(x-2)^2 = 4$

$x - 2 = \pm 2$

$x = 2 \pm 2$ → $4 = x$, → $0 = x$

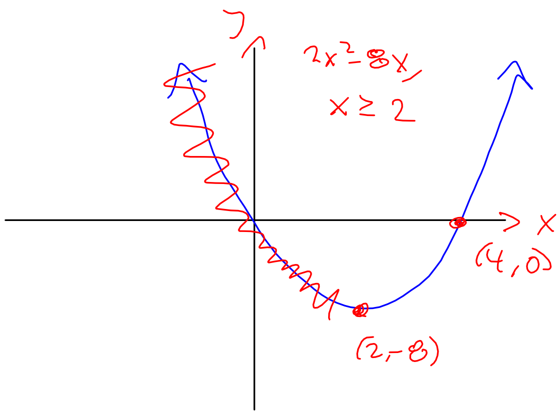
Also $y \geq -8$, from this step.

Now, $x \geq 2 \Rightarrow$

$x = 2 + \sqrt{\frac{y+8}{2}} \longrightarrow f^{-1}(x) = 2 + \sqrt{\frac{x+8}{2}}$

$\mathcal{D}(f) = \mathcal{R}(f^{-1}) = [2, \infty)$ GIVEN

$\mathcal{R}(f) = \mathcal{D}(f^{-1}) = [-8, \infty)$



In future

$$\int \frac{du}{\sqrt{u^2+6u}} = \int \frac{du}{\sqrt{(u+3)^2-9}} = \int \frac{dv}{\sqrt{v^2-9}}$$

$$u^2+6x = u^2+6u+3^2-9 = (u+3)^2-9$$

has a nice formula.

$$\int \frac{dx}{\sqrt{x^2+6x}} = \int \frac{dx}{\sqrt{(x+3)^2-9}} \qquad \begin{aligned} u &= x+3 \\ du &= dx \end{aligned}$$

$$= \int \frac{du}{\sqrt{u^2-9}}$$

Recall Circle

$x^2+y^2=r^2 \Longrightarrow$

$y^2=r^2-x^2 \Longrightarrow$

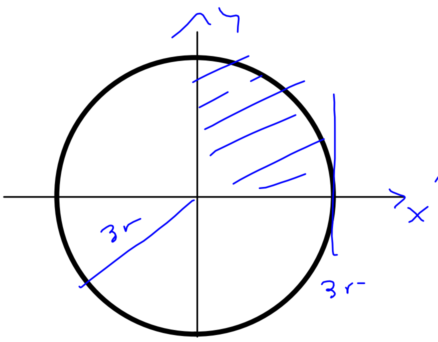
$y=\pm\sqrt{r^2-x^2}$

Top half $y=\sqrt{r^2-x^2}$

Bottom half $y=-\sqrt{r^2-x^2}$

So, changing vars in integral to "x"

$$2\pi\int_0^{3r} 16R\sqrt{9r^2-y^2}\,dy = 32\pi R\int_0^{3r}\sqrt{9r^2-x^2}\,dx$$



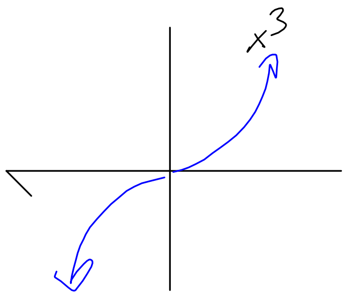
$\int_0^{3r}$

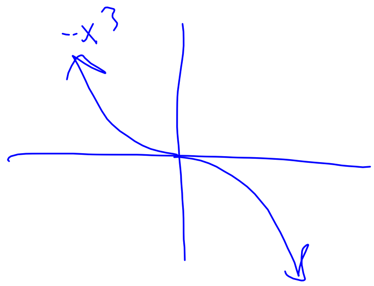
So $32\pi R\left[\frac{1}{4}\text{ Area of circle of radius } 3r\right]$

$= 32\pi R\left[\frac{1}{4}\right]\left[\pi(3r)^2\right]$

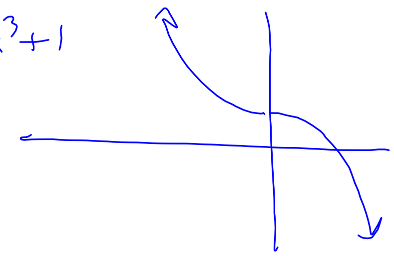
$= 8\pi R\,\pi(9r^2)$

$= 72\pi^2r^2R$? !

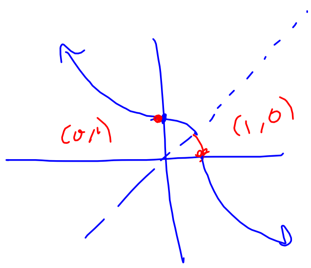




$1-x^3 = -x^3+1$



$\sqrt[3]{1-x^3}$



Part a set

$g(x) = \sqrt[3]{1-x^3} \Longrightarrow$

$g^{-1}(x) = \sqrt[3]{1-x^3} = g(x)$!

People from my class, last semester: I'll shoot you your graded final, etc, this wknd. Sorry didn't send, yet.
Scanned 'em last December.