Steve

MAT 202

CALC II

Techniques of Integration

and

Power Series -

Taylor Series

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^{n+1} \frac{x^{2n+1}}{(2n+1)!}$$

1 §6.1 Inverse Functions

1 §6.2 Exponential funcs -

2 §6.3 Logarithmic Funcs

By Wed.

Test 1, Ch 6,

in about 3 weeks

$\sum_{k=0}^{5}$ ~~~ will get you within 5 decimal places.

§6.1 1-to-1 functions and inverse functions.

1-to-1 : $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

Horizontal Line Test.

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

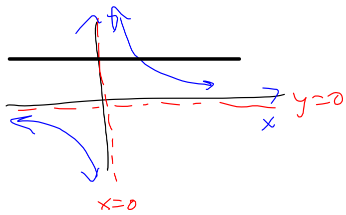
contrapositive.

A implies B is logically equivalent to NOT B implies NOT A.

$A \Rightarrow B$ has converse.

$B \Rightarrow A$

Is $\frac{1}{x}$ 1-to-1 ?



Prove $\frac{1}{x}$ is 1-to-1 :

Suppose $f(x_1) = f(x_2)$

Then $\frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_2 = x_1$

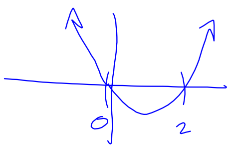
So it was the same point all along! $\therefore$ 1-to-1.

Proving A function is NOT 1-to-1 :

Find $x_1 \neq x_2 \ni f(x_1) = f(x_2)$

such that / so that.

Example $x^2 - 2x$ is not 1-to-1.



$x(x-2)$

$f(x) = x^2 - 2x$

Counter-example :

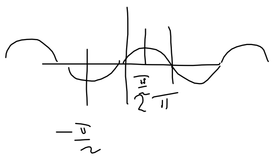
$x_1 = 0,\ x_2 = 2$, but $f(0) = f(2) = 0 \Rightarrow$

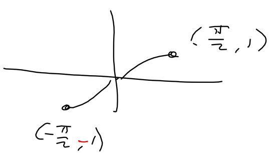
not 1-to-1,

If it's not 1-to-1, then the inverse relation is not a function.

What's our workaround ?

Restrict the domain.





Alex

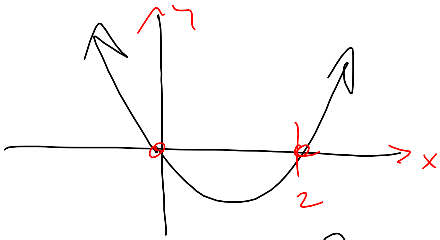
Restricted sine

$\mathcal{D} = [-\frac{\pi}{2}, \frac{\pi}{2}],\ \mathcal{R} = [-1, 1]$

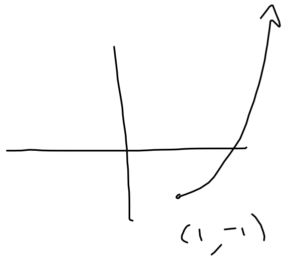
$\Rightarrow \sin^{-1}(x) = \arcsin(x)$ has

$\mathcal{D} = [-1, 1],\ \mathcal{R} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

So what about x^2-2x?



Restrict to $\mathcal{D} = [1, \infty)$



Now, to invert it

$$y^2-2y = x$$

$$y^2-2y+1^2 = x+1$$

$$(y-1)^2 = x+1$$

$$\sqrt{(y-1)^2} = \sqrt{x+1}$$

$$|y-1| = \sqrt{x+1}$$

$$y-1 = \pm\sqrt{x+1}$$

$$y = 1 \pm \sqrt{x+1}$$

which one's the inverse function?

$$\sqrt{x^2} = |x|$$

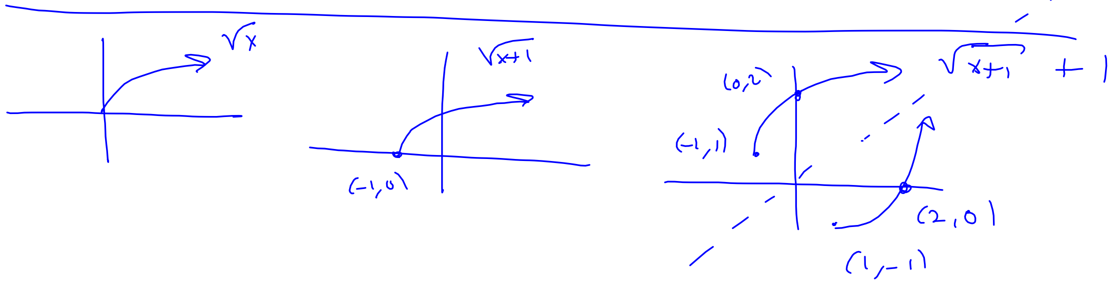
$$\sqrt{(-3)^2} = \sqrt{9} = 3$$

$$\sqrt{3^2} = \sqrt{9} = 3$$

$$|-3| = 3$$

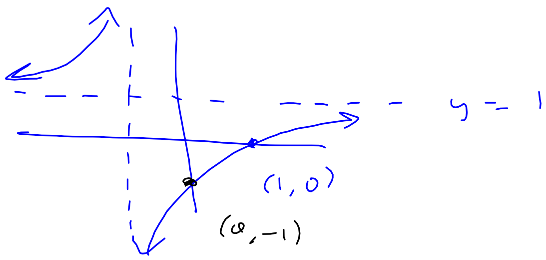
$$|3| = 3$$

So $f^{-1}(x) = 1+\sqrt{x+1}$



$f(x) = \frac{x-1}{x+1}$

$\frac{x}{x} = 1 = y$ is H.A.



$\frac{y-1}{y+1} = x$

$y - 1 = x(y+1) = xy + x$

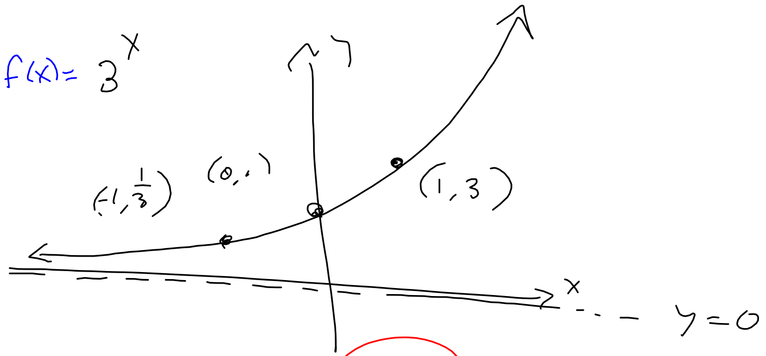
$y - xy = x + 1$

$y(1-x) = x + 1$

$y = \boxed{\frac{x+1}{1-x} = f^{-1}(x)}$

§6.2 Exponential Functions

$f(x) = 3^x$



$f(x) = b^x$ is cont's and smooth

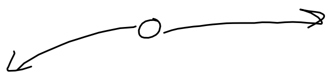
continuous

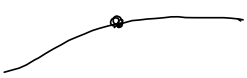
No breaks or holes

Continuous at $x=c$:

$$\lim_{x\to c} f(x) = f(c)$$

limit $\exists$ (exists)





But has hole.