Find the Taylor polynomial $T_3(x)$ for the function f centered at the number a.

$$f(x) = \frac{\ln(2x)}{3x}, \quad a = \frac{1}{2}$$

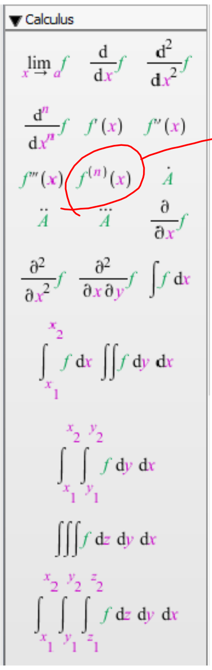
16. SCalc8 11.11.507.XP. (3353067)

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k \xrightarrow{\text{MAPLE}} sort\left(\sum_{k=0}^{n} \frac{f^{(k)}(a)\cdot(x-a)^k}{k!}\right)$$

$$f(x) = \frac{\ln(2x)}{3x}$$

$a = \frac{1}{2}$ → Can't do Maclaurin's, because $a=0$ makes division(s) by zero.

$n = 3$



$\frac{d^n f}{dx^n}$ = nth derivative!

MAPLE Palette for Calculus

MATHCAD

MATHEMATICA

Find the Taylor polynomial $T_3(x)$ for the function f centered at the number a.

$f(x) = \arcsin(3x), \quad a = 0$

15. SCalc8 11.11.506.XP. (3352689)

Find the Taylor polynomial $T_3(x)$ for the function f centered at the number a.

$f(x) = x + e^{-x}, \quad a = 0$

14. SCalc8 11.11.503.XP. (3352658)

$f(0) = 0$

$f'(0) = \frac{20\ m}{s}$

$f''(0) = \frac{4m}{s^2}$

$T_2(x) = 0 + 20x + \frac{4x^2}{2}$

$= 20x + 2x^2$

#13 (11.11.031)

Apr 29-8:50 AM

Use the Alternating Series Estimation Theorem to estimate the range of values of x for which the given approximation is accurate to within the stated error.

$\arctan(x) \approx x - \frac{x^3}{3} + \frac{x^5}{5}$ $\quad (|\text{error}| < 0.05)$ **12. SCalc8 11.11.029.** (3352912) Check your answer graphically. (Round your answers to three decimal places.)

11.11 #9

$f(x) = \ln(1+x)$

$f'(x) = \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} (-1)^n x^n \Rightarrow$

$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} + C$

want $\ln(1.4)$ w/in $\pm .01$

$f(0) = \ln(1+0) = 0 = C$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} + \ldots$

$T_2(.4) = .4 - \frac{.16}{2}$

want $\ln(1.4) = f(0.4)$

$= .4 - .08$

$a_{n+1} = (-1)^n \frac{x^{n+2}}{n+2}$

$= .32$

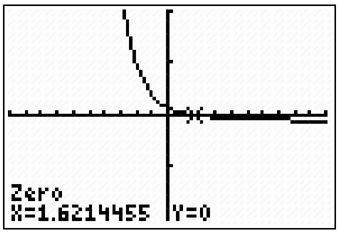
$|f(.4) - T_n(.4)| < .01$

$\underbrace{|f(.4) - T_n(.4)|}$

$|E_n| \le |a_{n+1}| = \left|(-1)^n \frac{.4^{n+2}}{n+2}\right| < .01$

$(-1)^n \frac{.4^{n+1}}{n+1}$

$.4^{n+2} < .01n + .02$



I solved

$.4^{n+2} - .01(n+2) = 0$

$\Rightarrow n \approx 1.6214455$

want this < 0 so $n = 2$ should do it.

The series IS alternating

$\left|\frac{x^3}{3}\right| < .01$

$\left|\frac{(.4)^2}{2}\right| = \frac{.16}{2} = .08$ want $< .01$

$\frac{.4^3}{3} = \frac{.048}{3} = .016$

$$\begin{array}{r} .048 \\ \times .4 \\ \hline .0192 \end{array}$$

$\frac{.4^4}{4} = \frac{.0192}{4} =$

$\frac{M}{(n+1)!}|x-a|^{n+1}$

$a = 0$

$x \in [-.4, .4]$

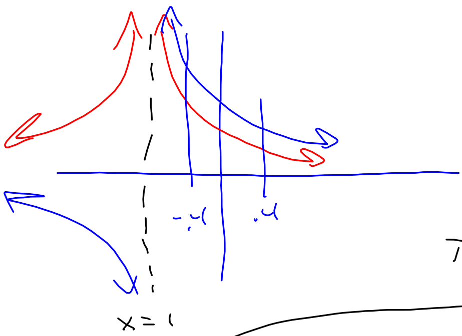
$f = \ln(1+x)$

$f^{(2)}(x) = -(1+x)^{-2}$

$f^{(1)} = \frac{1}{1+x} = (1+x)^{-1}$

$f^{(3)}(x) = 2(1+x)^{-3}$

Apr 29-9:18 AM



$$f^{(n)}(x) = \frac{(n-1)!}{(1+x)^n}$$

$$\underset{[-.4,.4]}{\text{Max}} |f^{(n)}(x)| = \frac{(n-1)!}{(.6)^n} \le M$$

Taylor Says $|E_n| \le \dfrac{M|x|^{n+1}}{(n+1)!}$

$$\text{“}=\text{''} \quad \frac{(n-1)!\,|x|^{n+1}}{(.6)^n (n+1)!} = \frac{|x|^{n+1}}{(n+1)(n)(.6)^n}$$

$$\frac{|-.4|^{n+1}}{n(n+1)(.6)^n} = \frac{.4\,|4\times10^{-1}|^n}{(n+1)(n)(6\times10^{-1})^n}$$

$$= \frac{.4\left(\frac{2}{3}\right)^n}{n(n+1)} < 0.01$$

$$\frac{\frac{2}{5}\left(\frac{2}{3}\right)^n}{n(n+1)} - .01 < 0$$

Taylor's Inequality will probably give you a better estimate.