MAT 202 Test 5, Spring 2018

100 Points Chapter 11 I Name___________________________

Answer up to 4 bonus questions for up to 20 bonus points. I suggest starting every problem and spending 2 minutes on it. Start a fresh piece of paper if you don't immediately finish a problem in 2 minutes.

1. Compute $\lim_{n\to\infty} a_n$ for each of the following a_n:

   a. (5 pts) $a_n = \dfrac{1}{n}$

   b. (5 pts) $a_n = \dfrac{\ln(n)}{n^{1/3}}$ Hint: Put this one in the hospital.

   c. (5 pts) $a_n = \dfrac{3n^2 - n}{\left(n^3 - \frac{1}{2}n^2\right)^2}$ $\quad$ Hint : $\left(n^3 + \text{smaller degree terms}\right)^2 = n^6 + \text{smaller degree terms.}$

2. State whether or not the series converges. Support your answer by the indicated criteria:

   a. Let $S = \sum_{n=1}^{\infty} \dfrac{1}{n}$

      i. (10 pts) Give this special series a name and what is well-known about this series.

      ii. (10 pts) Use Integral Test.

      iii. (5 pts bonus) Use Ratio Test. Hint: Conclude that the Ratio Test is inconclusive.

   b. Let $S = \sum_{n=1}^{\infty} \dfrac{\ln(n)}{n^{3/2}}$

      i. (10 pts) Use Integral Test.

      ii. (5 pts bonus) Use Direct Comparison. (Not as easy)

   c. Let $S = \sum_{n=1}^{\infty} \dfrac{3n^2 - n}{\left(n^3 - \frac{1}{2}n^2\right)^2}$

      i. (10 pts) Use Limit Comparison.

      ii. (5 pts bonus) Use the Integral Test. This is a little more challenging.

      iii. (5 pts bonus) Use Direct Comparison. This is more challenging.

d. Let $S=\sum_{k=1}^{\infty}(-1)^k \frac{1}{k}$

i. (10 pts) Use the Alternating Series Test

3. Let's delve more deeply into the sum $S=\sum_{k=1}^{\infty}(-1)^k \frac{1}{k}$.

a. (5 pts) Find a sufficient number n of terms in the partial sum S_n, so that the error $|R_n|<.001$, using the criterion for alternating series.

4. (10 pts) Let $S=\sum_{k=1}^{\infty}\frac{3^k}{k!}$. Show that S converges by any convincing method.

5. Let $S=\sum_{k=1}^{\infty}a_k=\sum_{k=1}^{\infty}\frac{1}{k^{3/2}}\approx 2.612375349$

a. (5 pts) Find the 4th partial S_4 for S. Round your final result to 4 places.

b. (5 pts) Let $f(n)=a_n$ and use the fact that $\int_{n+1}^{\infty} f(x)dx<|R_n|<\int_{n}^{\infty} f(x)dx$, and your result for S_4 in the previous problem, to improve your estimate of the sum S in the previous problem.

6. (10 pts) Compute the exact value of the (telescoping) sum $S=\sum_{n=1}^{\infty}\frac{2}{n^2+2n}=\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right)$

Bonus (5 pts bonus) Show that the surface area of a sphere of radius r = 3 is $4\pi r^2=4\pi(3)^2=36\pi$ by rotating the parametric curve given by the equations $x=3\cos(t), y=3\sin(t), 0\le t\le\pi$ about the x-axis. Full credit for writing the integral.