MAT 202 Test 4, Spring 2018

100 Points Chapter 10 Name____________________________

1. (10 pts) Consider the curve given by the parametric equations $x = t^2 - 2t,\ y = t + 3$. Sketch the curve by eliminating the parameter and. Plot enough points (say $t = 0,1,2,3$) to give you a direction of increasing *t*. Indicate the direction of increasing *t* with arrows.

2. (10 pts) Write the integral that gives the area under the parametric curve given by the equations $x = 3\cos(t), y = 3\sin(t)$, from $t = \frac{\pi}{6}$ to $t = \frac{5\pi}{6}$.

Bonus (5 pts) Evaluate the integral from #2.*

3. (10 pts) Show that the surface area of a sphere of radius $r = 3$ is $4\pi r^2 = 4\pi(3)^2 = 36\pi$ by rotating the parametric curve given by the equations $x = 3\cos(t), y = 3\sin(t), 0 \le t \le \pi$ about the *x*-axis. Full credit for writing the integral.

Bonus (5 pts) Evaluate the integral.*

4. Consider the parametric curve defined ty the parametric equations $x = t^2,\ y = t^3 - 18t$
   a. (10 pts) Find an equation of the tangent line to the parametric curve at $t = 3$.

   b. (10 pts) Eliminate the parameter and graph the curve.

   c. (10 pts) Find all points at which the tangent to the curve is horizontal. Label on graph.

   d. (10 pts) Find all points at which the tangent to the curve is vertical. Label on graph.

   e. **Bonus** (5 pts) Find all *t*-intervals on which the curve is concave down.*

5. We analyze the polar curve $r = 1 + 2\cos(2\theta)$.

   a. (10 pts) Check for symmetry.

   b. (10 pts) Sketch $r = 1 + 2\cos(2\theta)$ in *Cartesian* coordinates.

   c. (10 pts) Solve $\frac{dr}{d\theta} = 0$. This answer should be confirmed by the highs and lows of your Cartesian graph.

   d. (10 pts) Write the integral for the area bounded by the curve.

Bonus: (5 pts) Evaluate the integral for part 5d.*

Arc Length: $\int_a^b ds$

Chapter 8: $ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$, when $y = f(x)$ and $ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$, when $x = g(y)$

Chapter 10: $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$, when $x = x(t)$ is a function of t and $y = y(t)$ is a function of *t*.

$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\,d\theta$, when $r = r(\theta)$ is a function of θ, $x = r\cos\theta$ and $y = r\sin\theta$.

Surface of Revolution: $A = 2\pi\int_a^b y\,ds$ for rotations about the *x*-axis. $A = 2\pi\int_a^b x\,ds$ for rotation about the *y*-axis.

1st and 2nd Derivatives for Parametrics: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

Power-reduction: $\sin^2(x) = \frac{1-\cos(2x)}{2}$, $\cos^2(x) = \frac{1+\cos(2x)}{2}$

Symmetry:

Pole: Replace r by $-r$ θ by $\theta+\pi$ and obtain an equivalent equation

Polar Axis: Replace θ by $-\theta$ and obtain an equivalent equation.

The Line $\theta = \frac{\pi}{2}$: Replace θ by $\pi-\theta$ and obtain an equivalent equation.