Claim

$$\frac{n+5}{\sqrt{n-1}} \xrightarrow{n\to\infty} \infty$$

Scratch:

want $\frac{n+5}{\sqrt{n-1}} > M$

$n+5 > M\sqrt{n-1}$

$n^2+10n+25 \geq M^2(n-1) = M^2 n - M^2$

$n^2 + (10-M^2)n \geq -M^2-25$

$n^2+(10-M^2)n$

Making it too complicated

If $\frac{n+5}{\sqrt{n-1}} > M$, then

$n+5 > M$

$n > M-5$

Proof: Let $M>0$ be given

Try again. Prove something SMALLER $\to \infty$

$$\frac{n+5}{\sqrt{n-1}} > \frac{n}{\sqrt{n-1}} > \frac{n}{\sqrt{n}} = \sqrt{n}$$

Want $\sqrt{n} > M$

$n > M^2$

Proof:

Let $M>0$ be given. Define $N \equiv M^2$.

Let $n > N$. Then

$$\frac{n+5}{\sqrt{n-1}} > \frac{n}{\sqrt{n}} = \sqrt{n} > \sqrt{N} = \sqrt{M^2} = M \quad \blacksquare$$

Maclaurin's for §11.10

$f(x) = 2^x = f^{(0)}(x)$ $\qquad f^{(0)}(0) = 2^0 = 1$

$f'(x) = \ln(2) \cdot 2^x$ $\qquad f^{(1)}(0) = \ln(2)$

$f^{(2)}(x) = (\ln(2))^2 \cdot 2^x$ $\qquad f^{(2)}(0) = \ln(2)^2$

Find pattern for $f^{(n)}(0)$: $(\ln(2))^n$

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k = \sum_{k=0}^{\infty} \frac{(\ln(2))^k}{k!} x^k$$

$$2^x = e^{\ln(2^x)} = e^{x \ln(2)} = e^{(\ln(2))x}$$

$$\rightarrow \frac{d}{dx}\left[2^x\right] = \ln(2)\, e^{(\ln(2))x} = \ln(2)\, e^{\ln(2^x)} = \ln(2) \cdot 2^x$$

#19 Taylor's for

x^5+2x^3+x @ $a=2$

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(a)}{k!}(x-a)^k$$

$f^{(0)}(x) = x^5+2x^3+x$ $\quad$ $f^{(0)}(2) = 50$

$f^{(1)}(x) = 5x^4+6x^2+1$ $\quad$ $f^{(1)}(2) = 105$

$f^{(2)}(x) = 20x^3+12x$ $\quad$ $f^{(2)}(2) = 184$

$f^{(3)}(x) = 60x^2+12$ $\quad$ $f^{(3)}(2) = 252$

$f^{(4)}(x) = 120x$ $\quad$ $f^{(4)}(2) = 240$

$f^{(5)}(x) = 120$ $\quad$ $f^{(5)}(2) = 120$

$$f(x) = 50 + \frac{105}{1}(x-2) + \frac{184}{2!}(x-2)^2 + \frac{252}{3!}(x-2)^3 + \frac{240}{4!}(x-2)^4 + \frac{120}{5!}(x-2)^5$$

$$50 + 105(x-2) + 97(x-2)^2 + 42(x-2)^3 + 10(x-2)^4 + (x-2)^5$$

$\sum \frac{2}{n^2+1}$ converges

Direct.

$\frac{2}{n^2+1} < \frac{2}{n^2}$ & $2\sum \frac{1}{n^2}$ converges by p-test.

Limit Comparison

$\left|\frac{a_n}{b_n}\right| = \frac{2}{n^2+1} \cdot \frac{n^2}{2} = \frac{n^2}{n^2+1} = \frac{n^2}{n^2\left(1+\frac{1}{n^2}\right)} = \frac{1}{1+\frac{1}{n^2}}$

$\xrightarrow{n \to \infty} \frac{1}{1} = 1$

$\frac{2}{n^2-1} < \frac{2}{n^2 - \frac{n^2}{2}} = \frac{4}{n^2}$ & $\sum \frac{4}{n^2}$ $p=2$ passes.

$\sum \frac{\ln(n)}{n}$ diverges

$\frac{\ln(n)}{n} > \frac{1}{n}$ & $\sum \frac{1}{n}$ diverges by p-test.

$\sum \frac{\ln(n)}{n^2}$ converges

$\ln(n) < n^{\frac{1}{3}}$ (eventually),

& $\frac{n^{\frac{1}{3}}}{n^2} = \frac{1}{n^{\frac{5}{3}}}$ & $\sum \frac{1}{n^{5/3}}$ passes p-test.

$$\sum_{n=1}^{\infty}\left(\frac{5}{3}\right)^{-n+1} = \sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^{n-1} \quad \text{Converges}$$

$$\left(\frac{5}{3}\right)^{-n+1} = \frac{1}{\left(\frac{5}{3}\right)^{n-1}} = \frac{1}{\left(\left(\frac{3}{5}\right)^{-1}\right)^{n-1}}$$

$$= \frac{1}{\left(\left(\frac{3}{5}\right)^{n-1}\right)^{-1}} = \left(\frac{3}{5}\right)^{n-1}$$