http://harryzaims.com/202/old-tests/01-older-tests/test-final.pdf

Page 6 starts in with some good questions on series.

http://harryzaims.com/202/old-tests/01-older-tests/test-4-v01.pdf

Page 4 is where the Chapter 11 starts kicking in.

$$\lim_{n\to\infty}\left(\frac{1}{n}\right)^{\frac{1}{n}}$$

$$\frac{1}{n} \xrightarrow{n\to\infty} 0$$

$$\frac{1}{\infty}$$

$f(x) = x^{-2}$ $\boxed{a=1,}$ $n=2$ $(T_2(x))$

$.9 \leq x \leq 1.1$

(a) $T_2(x)$

$f^{(0)}(x) = x^{-2}$ $f^{(0)}(1) = \frac{1}{1^2} = 1$

$f^{(1)}(x) = -2x^{-3}$ $f^{(1)}(1) = -2(1) = -2$

$f^{(2)}(x) = 3\cdot 2\, x^{-4}$ $f^{(2)}(1) = 6 = 3\cdot 2$

$f^{(3)}(x) = -4\cdot 3\cdot 2\, x^{-5}$ $f^{(3)}(1) = -4\cdot 3\cdot 2$

$$T_2(x) = \sum_{k=0}^{2} \frac{f^{(k)}(1)}{k!}(x-1)^k = \frac{1}{0!}(x-1)^0 + \frac{-2}{1!}(x-1)^1 + \frac{6}{2!}(x-1)^2$$

$$= \boxed{1 - 2(x-1) + 3(x-1)^2 = T_2(x)}$$

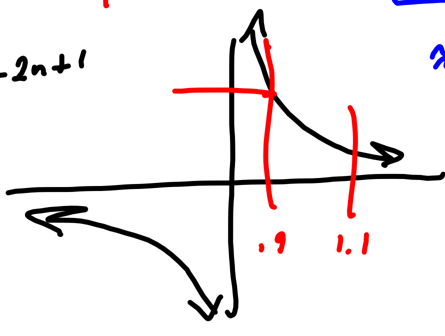
(b) $|R_n| \leq \frac{M}{(n+1)!}|x-1|^{n+1}$, where $M = \max_{[.9,\,1.1]} |f^{(n+1)}(x)|$

$n=2 \Rightarrow n+1=3$

$|f^{(3)}(x)| = |-24x^{-5}| = 24\left|\frac{1}{x^5}\right| \leq \boxed{24\left|\frac{1}{.9^5}\right| \equiv M}$

$\approx$ 40.64421074 $\approx M$

$*$ $\frac{1}{x^{2n-1}} = x^{-2n+1}$



$$|R_2| \leq \frac{(24)\left(\frac{10^5}{9^5}\right)}{3!}|x-1|^3$$

4

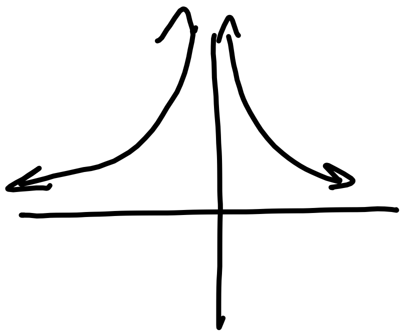
$$\leq \frac{(24)\left(\frac{10^5}{9^5}\right)}{6}|.1| = 4\left(\frac{10^5}{9^5}\right)\left(\frac{1}{10}\right) = 4\left(\frac{10}{9^5}\right)$$

$\approx$ 0.6774035123

$[.9, 1.1]$

$|R_n| \leq .6774035123$

$\frac{1}{x^{2n}}$



#9 11.9 P797

$$f(x) = \frac{x-1}{x+2} = \frac{x}{x+2} - \frac{1}{x+2} = x\left(\frac{1}{2+x}\right) - \frac{1}{2+x}$$

$$= x\left(\frac{1}{2(1+\frac{x}{2})}\right) - \frac{1}{2(1+\frac{x}{2})} = \frac{x}{2}\left[\frac{1}{1-(-\frac{x}{2})}\right] - \frac{1}{2}\left[\frac{1}{1-(-\frac{x}{2})}\right]$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

$$= \frac{x}{2}\sum_{k=0}^{\infty}\left(-\frac{x}{2}\right)^k - \frac{1}{2}\sum_{k=0}^{\infty}\left(-\frac{x}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} (-1)^k \left(\frac{x}{2}\right)^{k+1} + \sum_{k=0}^{\infty} (-1)^{k+1} \left(\frac{1}{2}\right)\left(\frac{x}{2}\right)^k$$

$$\left(-\frac{x}{2}\right)^k \left(\frac{x}{2}\right) = (-1)^k \left(\frac{x}{2}\right)^k \left(\frac{x}{2}\right) = (-1)^k \left(\frac{x}{2}\right)^{k+1}$$

$$\left(-\frac{1}{2}\right)\left(-\frac{x}{2}\right)^k = (-1)\left(\frac{1}{2}\right)(-1)^k \left(\frac{x}{2}\right)^k$$

$$= (-1)^{k+1}\left(\frac{1}{2}\right)\left(\frac{x}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left[\underline{(-1)^k \left(\frac{x}{2}\right)^{k+1}} + \underline{(-1)^{k+1} \left(\frac{1}{2}\right)\left(\frac{x}{2}\right)^k} \right]$$

$$= \sum_{k=0}^{\infty} \left[(-1)^k \left(\frac{x}{2}\right)^k \left[\frac{x}{2} + (-1)^1 \left(\frac{1}{2}\right) \right] \right]$$

$$\frac{(-1)^{k+1}}{(-1)^k} = (-1)^1$$

$$= \sum_{k=0}^{\infty} \left[(-1)^k \left(\frac{x}{2}\right)^k \left[\frac{x-1}{2} \right] \right]$$

Mean Trick

$$\frac{x-1}{x+2} = \frac{x+2-2-1}{x+2} = \frac{x+2-3}{x+2} = \frac{x+2}{x+2} - \frac{3}{x+2}$$

$$= 1 - \frac{3}{x+2}$$

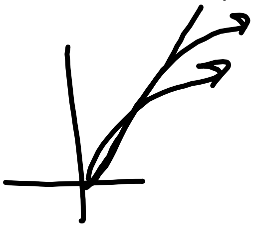
p-test intuition

$p = \frac{3}{4}$ diverges.

$$a_n = \frac{3}{n^{3/4}-n^{1/4}-1}$$

Limit Comparison $\frac{3}{n^{3/4}} = b_n$

$$\frac{a_n}{b_n} = \frac{3}{n^{3/4}-n^{1/4}-1}\cdot\frac{n^{3/4}}{3}$$



$$= \frac{\cancel{3}}{\cancel{n^{3/4}}\left(1-\frac{1}{n^{1/2}}-\frac{1}{n^{3/4}}\right)}\cdot\frac{\cancel{n^{3/4}}}{\cancel{3}} \xrightarrow{n\to\infty} 1$$

$0 \qquad 0$

$$\frac{n^{1/4}}{n^{3/4}} = \frac{1}{n^{\frac{3}{4}-\frac{1}{4}}}$$

Direct Comparison Bonus

$$\frac{1}{n^{3/4}-n^{1/4}-1} > \frac{1}{n^{3/4}-n^{1/4}} > \frac{1}{n^{3/4}}$$

& $\sum \frac{1}{n^{3/4}}$ Diverges by p-test.

Bigger Denominator
Smaller Fraction

So diverges by Direct Comparison.

$$\frac{1}{n^{3/4}+n^{1/4}+1} > \frac{1}{n^{3/4}+\frac{1}{4}n^{3/4}+\frac{1}{4}n^{3/4}} = \frac{1}{\frac{3}{2}n^{3/4}}$$

& $\sum \frac{1}{\frac{3}{2}n^{3/4}}$ diverges by $p = \frac{3}{4}$-test.

$$\sum \frac{1}{n^2-2n-1}$$

Limit Compare to $\frac{1}{n^2}$

Direct Comparison:

Show it's SMALLER than something that converges! NOT So Easy.

$$\frac{1}{n^2-2n-1} < \frac{1}{n^2-2\left(\frac{n^2}{8}\right)-\frac{n^2}{4}} = \frac{1}{n^2-\frac{n^2}{2}} = \frac{1}{\frac{n^2}{2}}$$

$$= \frac{2}{n^2} \quad \& \quad \sum \frac{2}{n^2} \text{ converges.}$$