Find $T_2(x)$ for $f(x) = \sqrt{x} = x^{\frac{1}{2}}$, $a = 4$

$f^{(0)}(4) = 4^{\frac{1}{2}} = 2$

$f^{(1)}(x) = \frac{1}{2}x^{-\frac{1}{2}}$ $\quad f^{(1)}(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4} =$

$f^{(2)}(x) = (-\frac{1}{2})(\frac{1}{2})x^{-\frac{3}{2}}$ $\quad f^{(2)}(4) = (-\frac{1}{2})(\frac{1}{2})\frac{1}{4^{3/2}} = -\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{8} = \frac{-1}{32}$

$$T_2(x) = \frac{f^{(0)}(4)}{0!}(x-4)^0 + \frac{f^{(1)}(4)}{1!}(x-4)^1 + \frac{f^{(2)}(4)}{2!}(x-4)^2$$

$$= \frac{2}{1} + \frac{\frac{1}{4}}{1}(x-4) + \frac{-\frac{1}{32}}{2}(x-4)^2$$

$$\boxed{= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 = T_2(x) \; @ \; x = 4}$$

Taylor says $|R_n| \leq \frac{M}{(n+1)!}|x-4|^{n+1}$

$= \frac{M}{3!}|x-4|^3$, where $M \geq \max|f^{(n+1)}(x)|$

New game: $f^{(2)}(x) = -\frac{1}{4}x^{-\frac{3}{2}}$

$f^{(n+1)}(x) = \left(-\frac{3}{2}\right)\left(-\frac{1}{4}\right)x^{-\frac{5}{2}}$, $\frac{1}{x^{\frac{5}{2}}} = \left(\sqrt{x}\right)^{-5}$

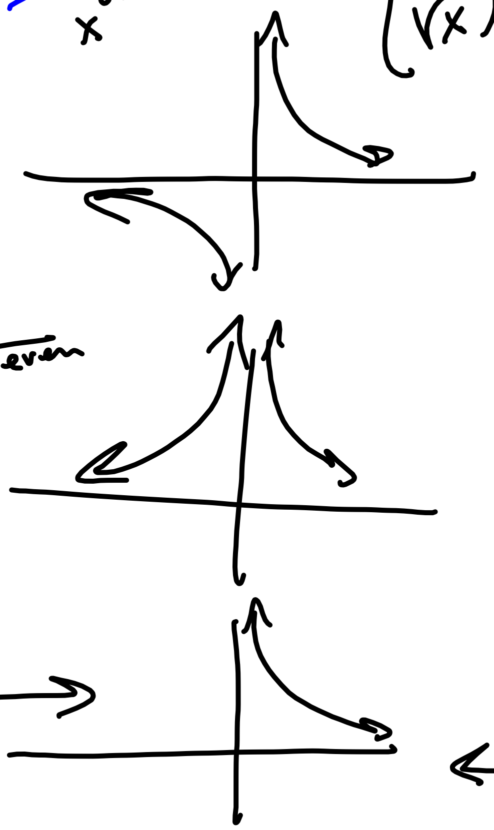
$\frac{3}{8}x^{-\frac{5}{2}} = \frac{3}{8\sqrt{x^5}}$

$\frac{1}{x^{odd}}$

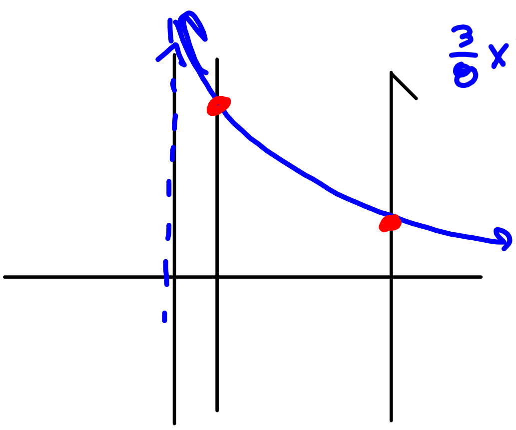
It's biggest to the left

What's the interval in question?

$\frac{1}{x^{even}}$



Let's just make it on [1,7]

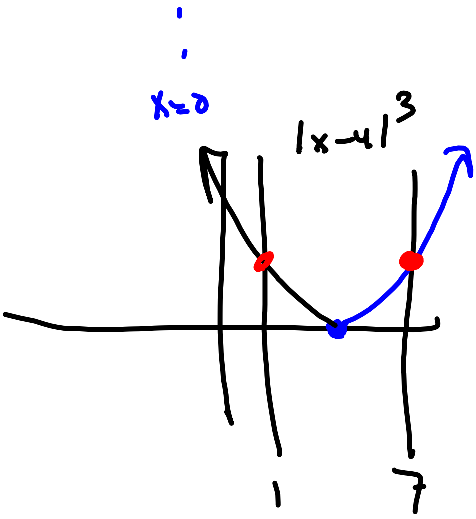


$\frac{3}{8}x^{-\frac{5}{2}}$

So $\max\limits_{[1,7]} \left| f^{(3)}(x) \right| = f^{(3)}(1)$

$= \frac{3}{8}(1)^{-\frac{5}{2}} = \boxed{\frac{3}{8} \equiv M}$

$|R_2| \le \frac{\frac{3}{8}}{}$ 3



$3!$ $|x-4|$

$|1-4|^3 = |-3|^3 = 27$

$|7-4|^3 = |3|^3 = 27$

$\leq \frac{\frac{3}{8}}{6}|27|$

$= \frac{3}{8} \cdot \frac{1}{\cancel{6}_{2}} \cdot \cancel{27}^{9} = \frac{27}{16}$

So we're within $\frac{27}{16}$ of $f(x)$ on $[1,7]$ with T_2.

Can you find Radius of convergence?

$\sqrt{\frac{a_{n+1}}{a_n}}$

Trouble is coming up with a general expression for a_n.

$a_0 = \frac{1}{2}$

$a_1 =$ we really need an expression for n^{th} derivative $f^{(n)}(x)$

$f(x) = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}}$

$f^{(2)}(x) = -\frac{1}{2}\cdot\frac{1}{2}x^{-\frac{3}{2}}$

$f^{(3)}(x) = (-\frac{3}{2})(-\frac{1}{2})(\frac{1}{2})x^{-\frac{5}{2}} = \frac{(3)(1)}{2^3}x^{-\frac{5}{2}}$

$2n-3$

$f^{(4)}(x) = (-\frac{5}{2})(-\frac{3}{2})(-\frac{1}{2})(\frac{1}{2})x^{-\frac{7}{2}} = \frac{5\cdot 3\cdot 1}{2^4}x^{-\frac{7}{2}}$

$2n-3$

$2n-5$

$2n-7$

$f^{(5)}(x) = \frac{7\cdot\boxed{5}\cdot 3\cdot 1}{2^5}x^{-\frac{9}{2}} = \frac{\boxed{(2n-3)}(2n-5)\cdots 3}{2^n} x^{-\frac{(2n-1)}{2}} = f^{(n)}(x)$

$f^{(n+1)}(x) = (2(n+1)-3)\boxed{(2(n+1)-5)}\cdots 3$

So, $\left|\frac{a_{n+1}}{a_n}\right| = \frac{(2(n+1)-3)}{(n+1)!}\cdot\frac{n!}{1}|x-4|$

$= \left|\frac{2n+2-3}{n+1}\right||x-4| = \frac{2n-1}{n+1}|x-4|$

$\xrightarrow{n\to\infty} 2|x-4| \overset{\text{want}}{<} 1$

$|x-4| < \frac{1}{2} = R$

$\Rightarrow -\frac{1}{2} < x-4 < \frac{1}{2}$

$\Rightarrow \frac{7}{2} < x < \frac{9}{2}$

Need to check

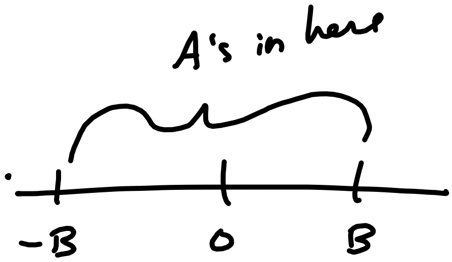
$x = \frac{7}{2}$ & $x = \frac{9}{2}$

Checking end points would be the next (difficult) step.

$$|x-4| < \frac{1}{2}$$

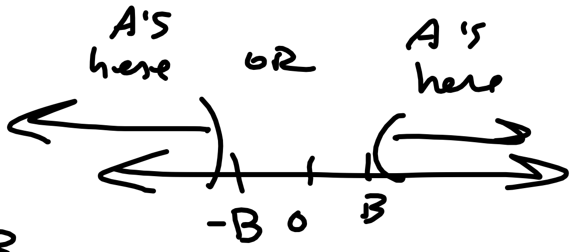
$x-4 < \frac{1}{2}$ AND $x-4 > -\frac{1}{2}$

$x < \frac{9}{2}$ and $x > \frac{7}{2}$



$|A| < B \rightarrow$

$A < B$ AND $A > -B$



$|A| > B \rightarrow$

$A > B$ OR $A < -B$