The command

5th-degree Taylor Polynomial for cot(x)

works perfectly on

http://www.wolframalpha.com/

$$\int \cot(x)\,dx = \int \frac{\cos x}{\sin x}\,dx = \int \frac{du}{u} = \ln|u| + C$$

$u = \sin x$
$du = \cos x\,dx$

$= \ln|\sin x| + C$ is not very helpful, Steve.
Too clever by half.

$$S = S_n + R_n$$
$$= \sum_{1}^{n} + \sum_{n+1}^{\infty}$$

Taylor's inequality gives us a bound on $|R_n|$

$$|S - S_n| = |R_n| \le \frac{M}{(n+1)!}|x-a|^{n+1}$$

$$M \ge \max|f^{(n+1)}(x)|$$

This is for $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(x)$

Taylor Series centered at $x=a$.

Sheer Butchery

I'll clean this up for Monday.

For now, skip to the last page, where I answered a question fairly well.

$f(x) = \sqrt{x}$, $a = 4$, $n = 2$

$= x^{\frac{1}{2}}$

$f(4) = \sqrt{4} = 2$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$

$f'(4) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$f^{(2)}(x) = f''(x) = -\frac{1}{4} x^{-\frac{3}{2}}$

$\frac{3}{2} = \frac{2n-1}{2}$

$f''(4) = -\frac{1}{4} \frac{1}{4^{3/2}} = -\frac{1}{4} \cdot \frac{1}{8} = -\frac{1}{32}$

$\vdots$

$f^{(n)}(x) =$

$4^{\frac{3}{2}} = \left(4^{\frac{1}{2}}\right)^3 = 2^3 = 8$

$$\sum_{k=0}^{2} \frac{f^{(k)}(4)}{k!} (x-4)^k = \frac{f^{(0)}(4)}{0!}(x-4)^0 + \frac{f^{(1)}(4)}{1!}(x-4)^1 + \frac{f^{(2)}(4)}{2!}(x-4)^2$$

$$\frac{2}{1} + \frac{\frac{1}{4}}{1!}(x-4)^1 - \frac{\frac{1}{32}}{2!}(x-4)^2$$

$$= 2 + \frac{1}{4}(x-4) + \frac{1}{64}(x-4)^2 = T_2(x)$$

$f(4) = 2 = T_2(4)$ ✓

what's the radius of convergence?

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{f^{(n+1)}(4)}{f^{(n)}(4)} \cdot \frac{(x-4)^{n+1}}{(x-4)^n} \right|$$

$n=1$ $\quad f^{(1)}(x) = -\frac{1}{2} x^{-\frac{1}{2}}$

$n=2$ $\quad f^{(2)}(x) = \left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) x^{-\frac{3}{2}}$

$\boxed{n=3}$ $\quad f^{(3)}(x) = \left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) x^{-\frac{5}{2}} = \boxed{-\frac{3}{2} x^{-1}} f^{(2)}(x)$

$\boxed{n=4}$ $\quad f^{(4)} = \left(-\frac{5}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right) x^{-\frac{7}{2}} = \boxed{-\frac{5}{2} x^{-1}} f^{(3)}(x)$

$\boxed{n=5}$ $\quad f^{(5)} = \underline{\left(-\frac{7}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{1}{2}\right)^2 x^{-\frac{9}{2}}}$

If this doesn't "pop"

$$f^{(5)} = -\frac{1}{4}\left(\frac{7\cdot 5\cdot 3}{2^3}\right) x^{-\frac{9}{2}}$$

$$f^{(4)} = \frac{1}{4}\left(\frac{5\cdot 3}{2^2}\right) x^{-\frac{7}{2}}$$

$$f^{(3)} = -\frac{1}{4}\left(\frac{3}{2^1}\right) x^{-\frac{5}{2}}$$

$n=5$ $\quad 7\cdot 5\cdot 3 = 3\cdot 5\cdot 7 = (2\cdot 1+1)(2\cdot 2+1)(2\cdot 3+1)$

$n=6$ $\quad 9\cdot 7\cdot 5\cdot 3 = (2\cdot 1+1)(2\cdot 2+1)(2\cdot 3+1)(2\cdot 4+1)$

$n=7$ $\quad 11\cdot 9\cdot 7\cdot 5\cdot 3 = (2\cdot 1+1)\cdots(2\cdot 5+1)$

$$f^{(n)}(x) = \frac{(2\cdot 1+1)(2\cdot 2+1)\cdots(2(n-2)+1)}{2^{n-2}} x^{-\frac{2n-1}{2}} =$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{\frac{((2\cdot 1+1)\cdots(2(n-2)+1)(2(n+1-2)+1)}{2^{n+1-2}} x^{-\frac{2(n+1)-1}{2}}}{\frac{(2\cdot 1+1)\cdots(2(n-2)+1)}{2^{n-2}} x^{-\frac{2n-1}{2}}}$$

$$= \left|\frac{2n-1}{2} x^{-1}\right| \quad \text{want} < 1$$

$$\frac{2n+2-1-(2n-1)}{2} = \frac{1+1}{2} = 1$$

$$\frac{x^{-\frac{2n+1}{2}}}{x^{-\frac{2n-1}{2}}}$$

$$2(n+1-2)+1$$
$$= 2(n-1)+1$$
$$2n-2+1$$
$$2n-1$$

$$= x^{-\frac{2n+1}{2} - \left(-\frac{2n-1}{2}\right)} = x^{-\left(\frac{2n+1-(2n-1)}{2}\right)}$$

$$= x^{-\frac{2}{2}} = x^{-1}$$

$$\frac{1}{x-1} = -\frac{1}{1-x} = -\sum_{k=0}^{\infty} x^k$$

$$\frac{1}{x-3} = -\frac{1}{3-x} = -\frac{1}{3(1-\frac{x}{3})} = -\frac{1}{3}\left(\frac{1}{1-x/3}\right)$$

$$= -\frac{1}{3}\sum_{k=0}^{\infty}\left(\frac{x}{3}\right)^k \quad = -\sum \frac{1}{3}\left(\frac{x}{3}\right)^k$$

$$-\sum_{k=0}^{\infty} x^k + \frac{1}{3}\frac{x^k}{3^k} = -\sum_{k=0}^{\infty}\frac{3^{k+1}+1}{3^{k+1}}x^k$$

$$x^k + \frac{1}{3}\cdot\frac{x^k}{3^k} = \frac{x^k}{1}\cdot\frac{3^{k+1}}{3^{k+1}} + \frac{x^k}{3^{k+1}}$$

$$= \frac{3^{k+1}x^k + x^k}{3^{k+1}} = \frac{(3^{k+1}+1)x^k}{3^{k+1}}$$