

$$\sum_{k=1}^{\infty} \frac{3^k}{k!} = \sum_{k=2}^{\infty} a_k$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{\frac{3^{n+1}}{(n+1)!}}{\frac{3^n}{n!}} = \frac{3^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n} = \frac{3}{n+1} \xrightarrow{n \rightarrow \infty} 0$$

$$= \quad (n+1)! = (n+1) \underbrace{(n)(n-1)\dots(3)(2)}_{n!}$$

$$\int x^{-1} dx = \int \frac{dx}{x} = \ln|x| + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad \text{if } n \neq -1.$$

$$\int x^{-\frac{3}{2}} dx = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C = -2x^{-\frac{1}{2}} + C$$

$$-\frac{3}{2} + 1 = -\frac{3}{2} + \frac{2}{2} = -\frac{1}{2}$$

$$\int_1^t \frac{\ln(x)}{x^{3/2}} dx = uv \Big|_1^t - \int_1^t v du$$

$$u = \ln x \quad dv = x^{-3/2} dx$$

$$du = \frac{1}{x} dx \quad v = -2x^{-1/2}$$

$$f(x) = \ln x$$

$$f'(x) = \frac{1}{x} = \frac{1}{1-1+x} = \frac{1}{1-(1-x)} = \sum_{k=0}^{\infty} (1-x)^k$$

$$\frac{1}{1-*} = \sum_{k=0}^{\infty} *^k = \frac{1}{1-*} \quad \text{if } -1 < * < 1$$

$$\text{So } \ln(x) = - \sum_{k=0}^{\infty} \frac{(1-x)^{k+1}}{k+1}$$

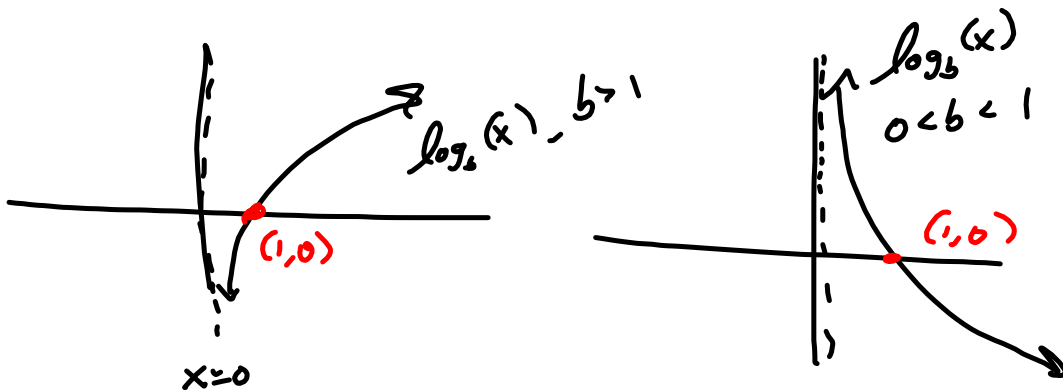
$$\int (1-x)^k dx = - \int (1-x)^k (-dx) = - \int u^k du =$$

$$= - \frac{u^{k+1}}{k+1} = - \frac{(1-x)^{k+1}}{k+1}$$

$u = 1-x$   
 $du = -dx$

$$= - \sum_{k=0}^{\infty} \frac{(-(x-1))^{k+1}}{k+1} = - \sum_{k=0}^{\infty} \frac{(-1)^{k+1} (x-1)^{k+1}}{k+1} = \sum_{k=0}^{\infty} \frac{(-1)^k (x-1)^{k+1}}{k+1}$$

$$= \frac{(x-1)^1}{1} - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + \dots$$



$$(x+1)^{2/3}$$

$$k=0$$

$$\binom{n}{k} = \frac{n(n-1)\dots(n-k+1)}{k!}$$

$$n-k+1 = n - (k-1)$$

$$\binom{2/3}{0} = 1$$

$$\binom{2/3}{1} = \frac{2/3}{1!} = \frac{2}{3}$$

$$\binom{2/3}{2} = \frac{\binom{2/3}{1}(-\frac{1}{3})}{2!} = -\frac{1}{9}$$

$$n - (k-1) = n-1 = \frac{2}{3} - 1 = -\frac{1}{3}$$

$$\binom{2/3}{3} = \frac{\binom{2/3}{2}(-\frac{1}{3})(-\frac{4}{3})}{3 \cdot 2} = \frac{2/27}{6} = \frac{2/27}{6} = \frac{2}{27} \cdot \frac{1}{6} = \frac{2}{162}$$

$$\frac{(-\frac{1}{9})(-\frac{4}{3})}{3} = \frac{4}{81}$$

$$\binom{2/3}{4} = \frac{4}{81} \cdot (-\frac{7}{3}) \left(\frac{1}{4}\right) = -\frac{7}{243}$$

$$\binom{2/3}{5} = \frac{\left(-\frac{7}{243}\right) \left(-\frac{10}{3}\right) \left(\frac{1}{5}\right)}{3} = \frac{14}{729}$$

$$(x+1)^{2/3} = 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 - \frac{7}{243}x^4 + \dots$$

$$(x+1)^{\frac{2}{3}} = 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 - \frac{7}{243}x^4$$

Taylor series:

$$f(0) = 1$$

$$f'(x) = \frac{2}{3}(x+1)^{-\frac{1}{3}}$$

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$f'(0) = \frac{2}{3}$$

$$f''(x) = -\frac{2}{9}(x+1)^{-\frac{4}{3}}$$

$$f''(0) = -\frac{2}{9}$$

$$f'''(x) = \frac{8}{27}(x+1)^{-\frac{7}{3}}$$

$$f'''(0) = \frac{8}{27}$$

$$f^{(4)} = -\frac{56}{81} = f^{(4)}(0)$$

$$f(x) = \frac{1}{0!} + \frac{\frac{2}{3}}{1!}x - \frac{\frac{2}{9}}{2!}x^2 + \frac{\frac{8}{27}}{3!}x^3 - \frac{\frac{56}{81}}{4!}x^4 + \dots$$

$$= 1 + \frac{2}{3}x - \frac{1}{9}x^2 + \frac{4}{81}x^3 - \frac{7}{243}x^4$$

See? It works!