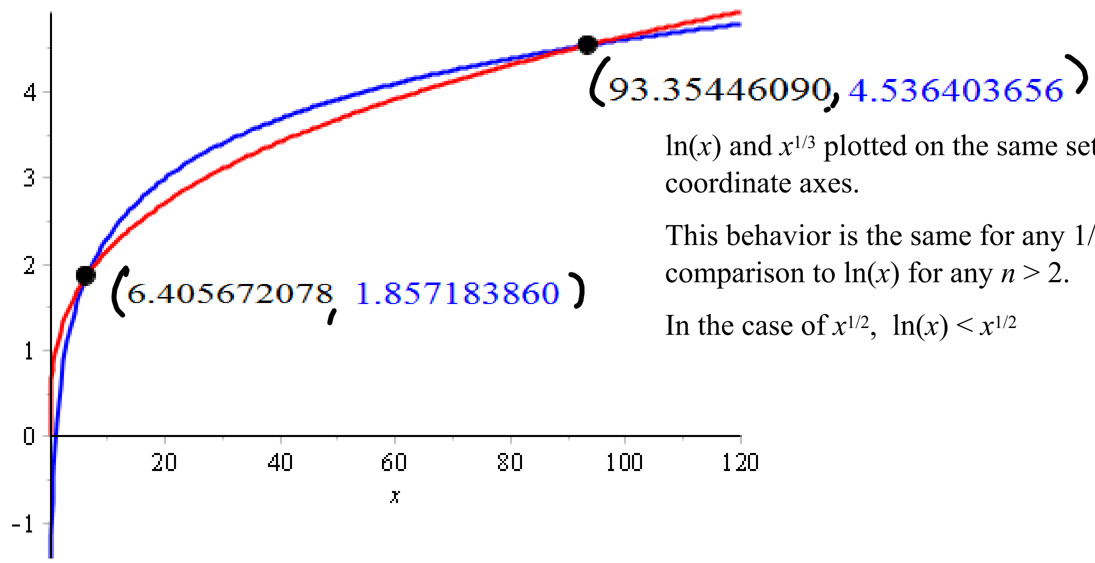

ln(x) and $x^{1/3}$ plotted on the same set of coordinate axes.

This behavior is the same for any 1/n power in comparison to ln(x) for any $n > 2$.

In the case of $x^{1/2}$, ln(x) < $x^{1/2}$

43–48 Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum (as in Example 7). If it is convergent, find its sum.

43. $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ **44.** $\sum_{n=1}^{\infty} \ln\frac{n}{n+1}$

$$\sum_{n=1}^{\infty} \frac{3}{n^2+3n} = \sum_{n=1}^{\infty}\left(\frac{1}{n} - \frac{1}{n+3}\right)$$

$$S_4 = \frac{1}{1} - \cancel{\frac{1}{4}} + \frac{1}{2} - \frac{1}{5} + \frac{1}{3} - \frac{1}{6} + \cancel{\frac{1}{4}} - \frac{1}{7} + \frac{1}{5} - \frac{1}{8}$$

$$S_6 = 1 - \cancel{\frac{1}{4}} + \frac{1}{2} - \cancel{\frac{1}{5}} + \frac{1}{3} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{4}} - \frac{1}{7} + \cancel{\frac{1}{5}} - \frac{1}{8} + \cancel{\frac{1}{6}} - \frac{1}{9}$$

$$= \frac{11}{6} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}$$

$n=6$

$$S_7 = 1 - \cancel{\frac{1}{4}} + \frac{1}{2} - \cancel{\frac{1}{5}} + \frac{1}{3} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{5}} - \frac{1}{8} + \cancel{\frac{1}{6}} - \frac{1}{9} + \cancel{\frac{1}{7}} - \frac{1}{10}$$

$$= \frac{11}{6} - \frac{1}{8} - \frac{1}{9} - \frac{1}{10}$$

$$= \frac{11}{6} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3} \quad *$$

$$S_n = \frac{11}{6} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3} \xrightarrow{n\to\infty} \frac{11}{6}$$

$a_n = n^{-\frac{4}{3}}$

I won't ask for 3-digit accuracy.

I'll ask "What's it take to get the error under .0001?"

Give me the best possible estimate for S', using the first 4 terms, S'_4.

$$S'_4 + \int_5^\infty x^{-\frac{4}{3}}dx \le S' \le S'_4 + \int_4^\infty x^{-\frac{4}{3}}dx$$

$$S'_4 = 1^{-4/3} + 2^{-4/3} + 3^{-4/3} + 4^{-4/3}$$

$\approx 1 + .396850263 + .231120425 + .157490131$

$\approx 1.785460819 \approx S'_4$

```
3^(-4/3)
         .231120425
4^(-4/3)
         .157490131
3^(-4/3)+Ans+.39
6850263+1
        1.785460819
```

$\int_5^\infty x^{-4/3}dx$ & $\int_4^\infty x^{-4/3}dx$

$$\int_5^t x^{-4/3}dx = -3x^{-\frac{1}{3}}\Big]_5^t = -3t^{-\frac{1}{3}} - (-3)(5)^{-\frac{1}{3}} \xrightarrow{t\to\infty}$$

$3\left(\frac{1}{\sqrt[3]{5}}\right)$

$$\int_4^\infty x^{-4/3}dx = 3\left(\frac{1}{\sqrt[3]{4}}\right)$$

$$\underbrace{\boxed{1.785460819}}_{S'_4} + \frac{3}{\sqrt[3]{5}} \le S' \le 1.785460819 + \frac{3}{\sqrt[3]{4}}$$

```
3^(-4/3)+Ans+.39
6850263+1
        1.785460819
Ans+3/5^(1/3)
        3.539871462
Ans+3/4^(1/3)
        5.429753037
```

$3.539871462 \le S' \le 5.429753037$

So, $S' \approx 3.607606928$

```
        3.153479179
3/4^(1/3)+3/5^(1
/3)+2*1.78546081
9
        7.215213856
Ans/2
        3.607606928
```

So, the error is $< \frac{5.42\ldots - 3.60\ldots}{2}$

(a, b)