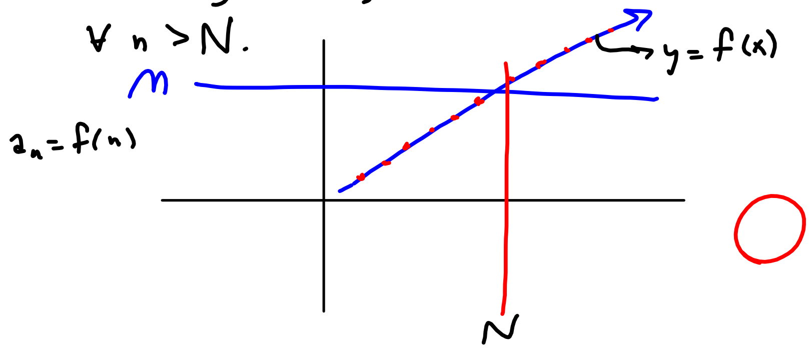
$\{a_n\} = \left\{\frac{n+5}{\sqrt{n-1}}\right\}$

$\frac{n}{\sqrt{n}} = \sqrt{n} \xrightarrow{n\to\infty} \infty$

Formally: $a_n \xrightarrow{n\to\infty} \infty$ means

given any $M>0$, $\exists N \in \mathbb{N} \ni a_n > M$ $\forall n > N$.



$M = 100$

want $\frac{n+5}{\sqrt{n-1}} > 100$

$n+5 > 100\sqrt{n-1}$

$(n+5)^2 > (100\sqrt{n-1})^2 = 10000(n-1)$

$n^2+10n+25 > a(n-1) = an-a$

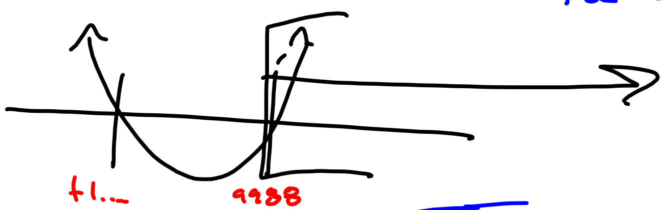
$n^2+10n-an+a+25 > 0$

$n^2+(10-a)n+a+25 > 0$

$n^2-9990n+10025 > 0$

$n^2-9990+4995^2 - 24950025 + 10025 > 0$

$(n-4995)^2 - 24940000 = 0$ ANALYZE For >0



$n-4995 = \pm\sqrt{24940000}$

$n = 4995 \pm \sqrt{24940000}$ 1.--

$n \approx 4995 \pm 4993.996396$ 9988.996396

Let $\boxed{N = 9989}$

N = 9998 makes any $n > N$ work.

$n+5 > 100\sqrt{n-1}$

$(n+5)^2 > (100\sqrt{n-1})^2 = 10000(n-1)$

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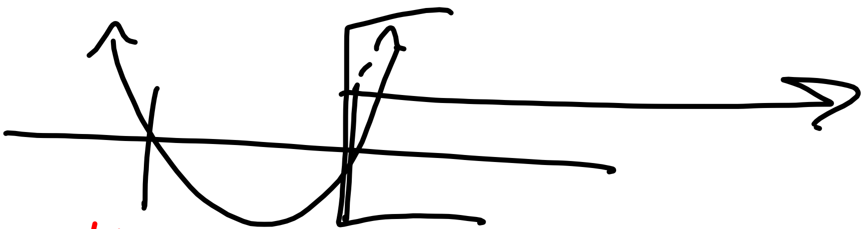
$n^2-9990+4995^2 -$ 24950025 $+10025 > 0$

$(n-4995)^2 -$ 24940000 $= 0$ ANALYZE FOR >0



+1... 9988

$n-4995 = \pm\sqrt{24940000}$

$n = 4995 \pm \sqrt{24940000}$ 1...

$n \approx 4995 \pm$ 4993.996396 9988.996396

Let $\boxed{N = 9989}$

N= 9998 makes any n > N work.

$$\sum \frac{2}{n^2+1}$$

$\frac{2}{n^2+1} < \frac{2}{n^2}$ Direct comparison converges by p-test.

$\sum \frac{2}{n^2-1}$ converges for pretty much same p-test reasons. but direct comparison's a pain in the ass.

$\frac{2}{n^2-\frac{n^2}{2}}$ is the trick, then.

b/c $\frac{2}{\frac{n^2}{2}} > \frac{2}{n^2-1}$

Limit comparison works more often.

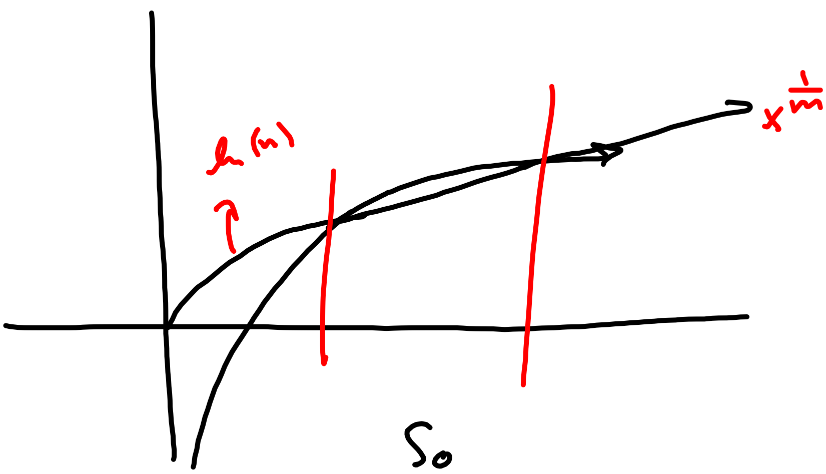
$$\left|\frac{a_n}{b_n}\right| = \left|\frac{\frac{2}{n^2-1}}{\frac{2}{n^2}}\right| = \left|\frac{n^2}{n^2-1}\right| \xrightarrow{n\to\infty} 1$$

$$\frac{5n^2-27n+11}{3n^3+5n^2} \xrightarrow{n\to\infty} 0$$

$$\frac{n^2\left(5-\frac{27}{n}+\frac{11}{n^2}\right)}{n^3\left(3+\frac{5}{n}\right)} = \frac{5-\frac{27}{n}+\frac{11}{n^2}}{n\left(3+\frac{5}{n}\right)}$$

$\frac{27}{n} \to 0$, $\frac{11}{n^2} \to 0$, $\frac{5}{n} \to 0$

$\ln(n) < n^{\frac{1}{m}}$, eventually, for ANY m.



So

$$\sum \frac{\ln(n)}{n^{1.1}} = \sum \frac{\ln(n)}{n^{11/10}}$$

$$n^{\frac{11}{10}-\frac{1}{20}} = n^{\frac{22-1}{20}} = n^{\frac{21}{20}}$$

$$\frac{\ln(n)}{n^{\frac{11}{10}}} < \frac{n^{\frac{1}{20}}}{n^{\frac{11}{10}}} = \frac{1}{n^{\frac{21}{20}}}$$

$$\sum_{n=1}^{\infty}\left(\frac{5}{3}\right)^{-n+1} = \sum\left(\frac{5}{3}\right)^{-1(n-1)} = \sum\left(\left(\frac{5}{3}\right)^{-1}\right)^{n-1}$$

$$= \sum_{n=1}^{\infty}\left(\frac{3}{5}\right)^{n-1} = \frac{1}{1-\frac{3}{5}} = \frac{5}{2}$$

$$S_n = 2\left(\frac{1-r^n}{1-r}\right)$$

$$\sum \frac{5^n}{n!}$$

$$\frac{\frac{5^{n+1}}{(n+1)!}}{\frac{5^n}{n!}} = \frac{5^{n+1}}{(n+1)n!}\cdot\frac{n!}{5^n} = \frac{5}{n+1} \xrightarrow{n\to\infty} 0$$

$$\sum_{n=1}^{\infty}\left(\frac{11n^3+\cdots}{5n^3+\cdots}\right)^n$$

$$a_n=\left(\frac{11n^3+\cdots}{5n^3+\cdots}\right)^n \xrightarrow{n\to\infty} \left(\frac{11}{5}\right)^{\infty}$$

$$\sqrt[n]{a_n} = \frac{11n^3+\cdots}{5n^3+\cdots} \xrightarrow{n\to\infty} \frac{11}{5} > 1$$

Diverges.

$$S = \sum_{n=1}^{\infty} \frac{1}{n^{4/3}}$$

$$S_6 = 1 + \frac{1}{2^{4/3}} + \frac{1}{3^{4/3}} + \frac{1}{4^{4/3}} + \frac{1}{5^{4/3}} + \frac{1}{6^{4/3}}$$

$$|R_6| \le \int_6^{\infty} \frac{dx}{x^{4/3}} = \int_6^{\infty} x^{-\frac{4}{3}} dx .$$

$$\int_6^{t} x^{-\frac{4}{3}} dx = -3x^{-\frac{1}{3}} \Big]_6^t = -3t^{-\frac{1}{3}} - (-3(6)^{-\frac{1}{3}})$$

$$\xrightarrow{t \to \infty} 3(6)^{-\frac{1}{3}} = \frac{3}{\sqrt[3]{6}} = \frac{3\sqrt[3]{36}}{6}$$

$$S = S_6 + R_6$$

$$S_6 + \int_7^{\infty} f(x)dx \le S \le S_6 + \int_6^{\infty} f(x)dx$$

$$f(x) = x^{-\frac{4}{3}}$$