§11.5 Alternating Series.

S $a_k \geq 0 \quad \forall\, k = 0, 1, \ldots$

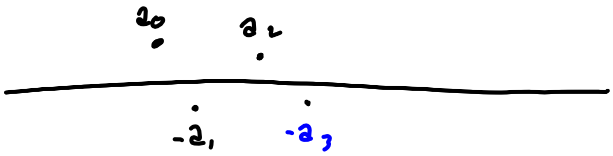
Then $\sum_{k=0}^{\infty} (-1)^k a_k$ is an alternating series.

$a_0 - a_1 + a_2 - a_3 + \cdots$

It converges if $a_n \xrightarrow{n \to \infty} 0$,

provided $a_{n+1} < a_n < \ldots$

what's it converge to? nothing here.



Alternating Series Estimation Theorem If $s = \sum (-1)^{n-1} b_n$, where $b_n > 0$, is the sum of an alternating series that satisfies

$$\text{(i) } b_{n+1} \leqslant b_n \qquad \text{and} \qquad \text{(ii) } \lim_{n \to \infty} b_n = 0$$

then $$|R_n| = |s - s_n| \leqslant b_{n+1}$$

$$|R_n| = |S - S_n| = |b_{n+1} - b_{n+2} + b_{n+3} - \cdots|$$

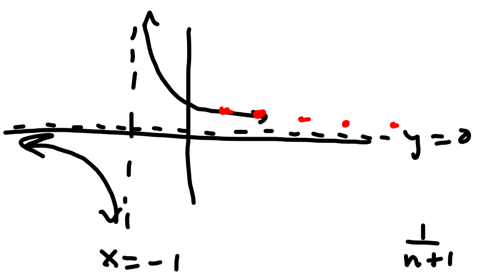
$$\leq b_{n+1}$$

Does b_n converge to zero?

Are b_n's all pos.?

Are b_n's decreasing?

$b_n = \frac{1}{n+1}$ is this decreasing?



$f(x) = \frac{1}{x+1} = (x+1)^{-1}$

$f'(x) = -(x+1)^{-2} = -\frac{1}{(x+1)^2}$

< 0 Decreasing.

$\frac{1}{n+1} > 0$ when $n \geq 0$

$\frac{1}{n+1} \xrightarrow{n \to \infty} 0$

Converges.

Harmonic Series $\sum \frac{1}{n}$ diverges

$$\sum (-1)^n \left(\frac{1}{n}\right) \text{ converges}$$

$$\sum (-1)^n \left(\frac{1}{\sqrt[15]{n}}\right)$$

conditional convergence

We say Alternating series converge conditionally, as opposed to converging absolutely.

Absolute convergence of $\sum a_n$ means $\sum |a_n|$ converges.

$\sum a_n = \sum (-1)^n \frac{1}{n}$ converges conditionally, but not absolutely.

because $\sum |a_n| = \sum \frac{1}{n}$ diverges.

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