S' 11.3 #38
11.4 #

① Intuition says whether it converges/diverges

Direct Comparison
Converges: Find something bigger that converges
Diverges: " " smaller " diverges

Limit Comparison is more powerful.

$\lim \frac{a_n}{b_n} = c \in \mathbb{R}$ says a_n & b_n have the same convergence properties

$c = 0$ a_n converges "better"

$c = \infty$ b_n converges "better"

11.3

38 $\sum_{n=1}^{\infty} ne^{-2n}$ to 4 decimal places.

$R_n < .00005$ to be accurate to 4 places.

$$R_n \le \int_n^\infty xe^{-2x}dx \le .00005 = \frac{5}{100{,}000}$$

$$\Rightarrow \int_n^t xe^{-2x}dx = uv - \int v\,du = -\tfrac{1}{2}xe^{-2x}\Big|_n^t - \int_n^t -\tfrac{1}{2}e^{-2x}dx$$

$u = x$ $\quad dv = e^{-2x}dx$

$du = dx$ $\quad v = -\frac{1}{2}e^{-2x}$

$$= -\tfrac{1}{2}te^{-2t} - \left(-\tfrac{1}{2}ne^{-2n}\right) + \tfrac{1}{2}\int \left(-\tfrac{1}{2}\right)e^{-2x}(-2dx)$$

$u = -2x$

$du = -2dx$

$$= -\tfrac{1}{2}te^{-2t} + \tfrac{1}{2}ne^{-2n} - \tfrac{1}{4}e^{-2x}\Big|_n^t$$

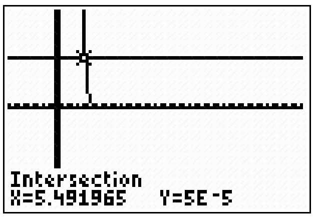
$$= -\tfrac{1}{2}te^{-2t} + \tfrac{1}{2}ne^{-2n} - \tfrac{1}{4}e^{-2t} - \left(-\tfrac{1}{4}e^{-2n}\right)$$

$$\xrightarrow{t\to\infty} \tfrac{1}{2}ne^{-2n} + \tfrac{1}{4}e^{-2n} < .00005$$

$$\tfrac{1}{4}e^{-2n}(2n+1) < \frac{5}{100000}$$

No nice analytic solution. NEED TECH.

Solve this digitally.



We obtain $n \approx 5.491965$ when it $\approx .00005$

So $n = 6$ does it.

wolframalpha.org

" Solve $\frac{1}{4}e^{-2n}(2n+1) = 5/100000$

$$\sum_{k=0}^{\infty} ar^k = a\left(\frac{1}{1-r}\right) \quad \text{if } |r|<1$$

So $\frac{1}{1-x} = 1+x+x^2+x^3+\cdots$ for $|x|<1$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = 1+(-x^2)+(-x^2)^2+(-x^2)^3+\cdots$$
$$= 1-x^2+x^4-x^6+x^8-x^{10}\cdots,$$

We get a <u>TON</u> of milage out of GEOMETRIC SERIES.

Suppose $f(x)$ <u>HAS</u> a power series.

$$f(x)=\sum_{k=0}^{\infty} c_k x^k = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

$$f(0) = c_0 + c_1(0) + c_2(0)^2 + \cdots = c_0$$

$$f'(x) = c_1 + 2c_2 x^1 + 3c_3 x^2 + 4c_4 x^3 + \cdots$$

$$f'(0) = c_1 + 0 + 0 + 0\cdots = c_1$$

$$f''(x) = 2c_2 + 3\cdot 2\, c_3 x + 4\cdot 3\, c_4 x^2 + \cdots$$

$$f''(0) = 2c_2$$

$$f'''(x) = 3\cdot 2c_3 + 4\cdot 3\cdot 2c_4x + 5\cdot 4\cdot 3c_5x^2$$

$$f'''(0) = 3\cdot 2c_3 = 3!\,c_3$$

$$f^{(iv)}(0) = 4\cdot 3\cdot 2c_4 = 4!\,c_4$$

$$f^{(v)}(0) = 5\cdot 4\cdot 3\cdot 2c_5 = 5!\,c_5$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

Maclaurin's Series for $f(x)$

$$c_0 = f(0)$$

$$c_1 = f'(0)$$

$$c_2 = \frac{f''(0)}{2} = \frac{f''(0)}{2!}$$

$$c_3 = \frac{f'''(0)}{3\cdot 2} = \frac{f^{(3)}(0)}{3!}$$

$$\vdots$$

$$c_n = \frac{f^{(n)}(0)}{n!}$$

Taylor's

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

$f(x) = \cos(x)$ Maclaurin Series

$f(0) = 1 = c_0$

$f'(x) = -\sin(x)$ $f'(0) = 0 =$

$f''(x) = -\cos(x)$ $f''(0) = -1$

$f'''(x) = \sin(x)$ $f'''(0) = 0$

$f^{(4)}(x) = \cos x$ $f^{(4)}(0) = 1$

$c_0 = 1,\ c_1 = 0,\ c_2 = \frac{f''(0)}{2!} = \frac{1}{2},\ c_3 = 0,\ c_4 = \frac{f^{(4)}(0)}{4!}$

$n=0$ $n=1$ $n=2$ $n=3$

$1x^0 - \frac{1x^2}{2!} + \frac{1x^4}{4!} - \frac{1x^6}{6!}$ $a_n = (-1)^n \frac{x^{2n}}{(2n)!}$

$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $\frac{x^n}{n!} \xrightarrow{n \to \infty} 0$

$= 1 - \frac{1}{2}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 + \cdots -$

$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$ Most

$S_n = \sum_{k=0}^{n} c_k (x-c)^k$ is most accurate on a neighborhood of $x = c$.