§11.2

$$\sum_{n=1}^{\infty} \frac{6(2^{2n-1})}{3^n} : \qquad \frac{6(4^n)(\frac{1}{2})}{3^n} = \frac{3\cdot 4^n}{3^n} = \frac{4^{n-1}\cdot 4}{3^{n-1}}$$

$$= \sum_{k=1}^{\infty} \left(\frac{4}{3}\right)^{n-1}\cdot 4 = 4\sum_{k=1}^{\infty}\left(\frac{4}{3}\right)^{n-1}$$

Diverges. Geometric $\sum_{k=1}^{\infty} ar^{k-1}$ where $r = \frac{4}{3} > 1$.

$$\sum_{k=1}^{\infty} ar^{k-1} = \sum_{k=0}^{\infty} ar^{k}$$

$$2^{2n-1} = 2^{2n}\cdot 2^{-1} = (2^2)^n(2^{-1})$$
$$= (4^n)\left(\frac{1}{2}\right)$$

11.2 #28 $\quad \frac{1}{3} + \frac{2}{9} + \frac{1}{27} + \frac{2}{81} + \frac{1}{243} \cdots$

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{81} + \frac{1}{243} \cdots$$

$$= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \cdots$$
$$+ \frac{1}{9} + \frac{1}{81} + \cdots$$

If re-arrangement is legit (Questionable)

Then it's $\sum_{k=1}^{\infty} \left(\frac{1}{3}\right)^n + \sum_{k=1}^{\infty}\left(\frac{1}{9}\right)^n$

$$= \sum_{k=1}^{\infty}\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{n-1} + \sum_{k=1}^{\infty}\left(\frac{1}{9}\right)\left(\frac{1}{9}\right)^{n-1}$$

$$= \frac{1}{3}\left(\frac{1}{1-\frac{1}{3}}\right) + \frac{1}{9}\left(\frac{1}{1-\frac{1}{9}}\right)$$

$$= \frac{1}{3}\left(\frac{3}{2}\right) + \frac{1}{9}\left(\frac{9}{8}\right) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$$

Comparison & Limit comparison Tests.

$a_n > 0$, $f'(n) < 0$

$\sum a_n$ has positive decreasing (eventually) terms.

If $a_n < b_n$ & $\sum b_n$ converges, then $\sum a_n$ converges.

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3} = \sum a_n.$$

Let $b_n = \frac{n}{n^3} = \frac{1}{n^2}$ & $\sum b_n$ converges by p-test.

$$\sum_{n=1}^{\infty} \frac{n-1}{n^3+7}$$

$b_n = \frac{n}{n^3} = \frac{1}{n^2}$ &

But Comparison Test Sucks for something like $\sum \frac{n+1}{n^3-7}$ b/c it's not obvious how to find a $b_n > a_n$ where $\sum b_n$ converges, although your intuition should scream $b_n = \frac{1}{n^2}$

Analysis Trickery:

At worst, this is bonus.

$$\frac{n+1}{n^3-7} < \frac{n+\frac{n}{2}}{n^3-\frac{n^3}{2}} = \frac{\frac{2n+n}{2}}{\frac{2n^3-n^3}{2}} = \frac{\frac{3n}{2}}{\frac{n^3}{2}} = \frac{3n}{n^3} = \frac{3}{n^2}$$

$$\sum \frac{1}{n^2} = \frac{\pi^2}{6}$$

what you REALLY want is
Limit Comparison Test

$\sum a_n$ converges if $\sum b_n$ converges and

$$\frac{a_n}{b_n} \xrightarrow{n\to\infty} c > 0.$$

$$\sum \frac{n+1}{n^3-7} = \sum a_n \qquad \sum b_n = \sum \frac{1}{n^2}$$

$$\frac{a_n}{b_n} = \frac{\frac{n+1}{n^3-7}}{\frac{1}{n^2}} = \frac{n+1}{n^3-7}\cdot\frac{n^2}{1} = \frac{n^3+n^2}{n^3-7} \xrightarrow{n\to\infty}$$

$$\frac{n^3+n^2}{n^3-7} = \frac{\cancel{n^3}\left(1+\frac{1}{n}\right)}{\cancel{n^3}\left(1-\frac{7}{n^3}\right)} \xrightarrow{n\to\infty} 1$$

$\frac{1}{n} \to 0$

$\frac{7}{n^3} \to 0$

If $a_n > b_n$ & $\sum b_n$ diverges, then so does $\sum a_n$.

$$\sum a_n = \sum \frac{n+7}{3n^2-4n+5}$$

Looks like $\frac{n}{3n^2} = \frac{1}{3n}$

$= \frac{1}{3n} = \frac{1}{3} \cdot \frac{1}{n}$

I think it's pretty harmonic.

$\sum b_n = \sum \frac{1}{n}$ diverges.

So compare: $\frac{a_n}{b_n} = \frac{\frac{n+7}{3n^2-4n+5}}{\frac{1}{n}}$

$= \frac{n^2+7n}{3n^2-4n+5} \xrightarrow{n \to \infty} \frac{1}{3}$ So the $\sum a_n$ diverges.