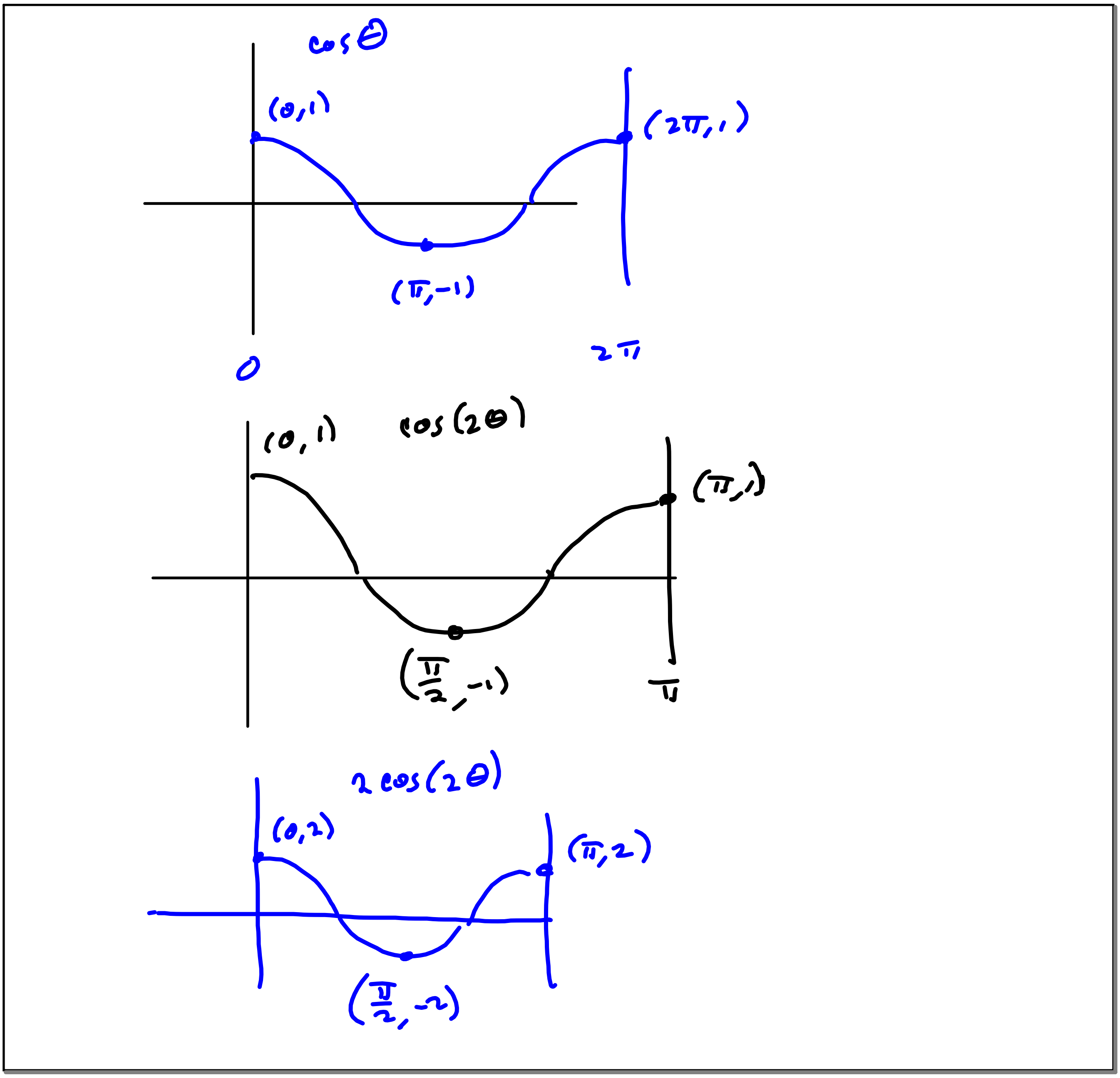
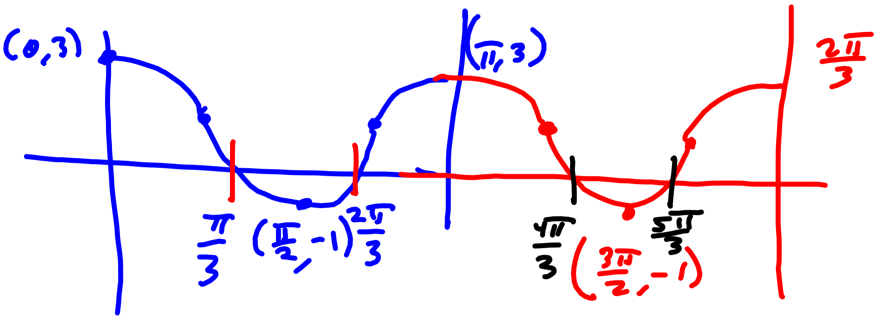

cos θ
(0,1)
(2π,1)
(π,−1)
0
2π
cos(2θ)
(0,1)
(π,1)
(π/2,−1)
π
2cos(2θ)
(0,2)
(π,2)
(π/2,−2)

$2\cos(2\theta)+1$

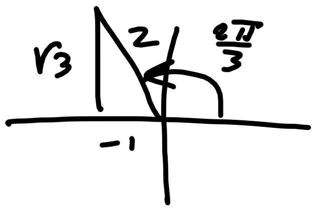


$0 \le \theta \le 2\pi$ to go around, so

$0 \le 2\theta \le 4\pi$ for solutions of

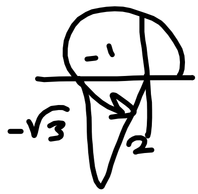
$1+2\cos(2\theta)=0$ $\qquad\qquad \cos(2\theta)=-\frac{1}{2}$

$2\cos(2\theta)=-1$

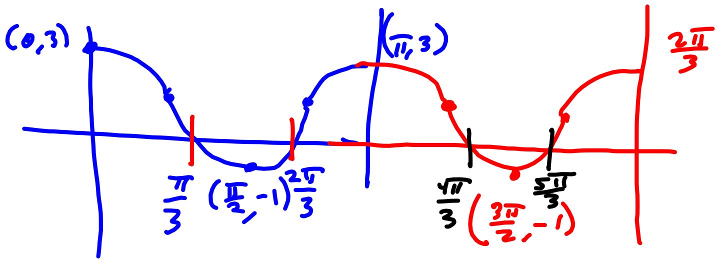


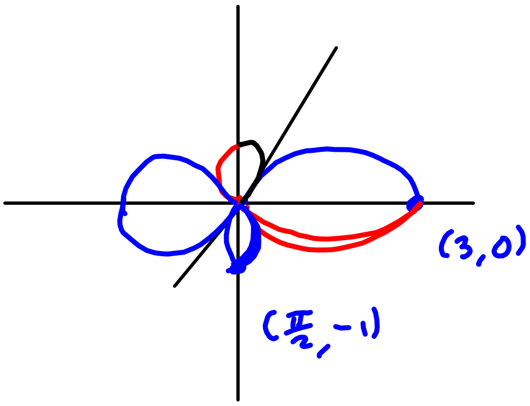
$\cos(2\theta)=-\frac{1}{2}$

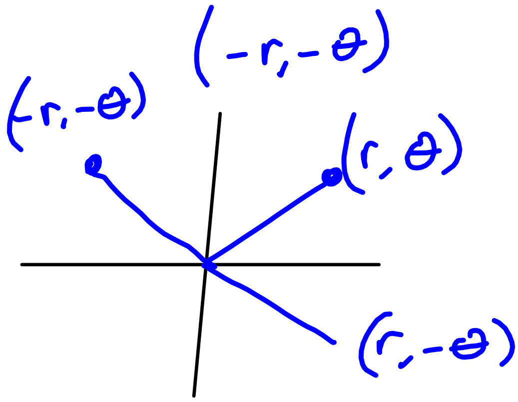
$2\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

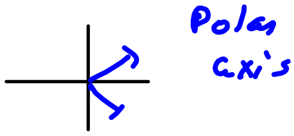


$\theta = \frac{2\pi}{6}=\frac{\pi}{3}, \frac{4\pi}{6}=\frac{2\pi}{3}$









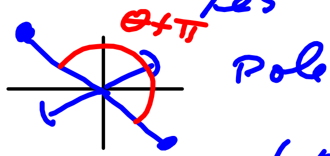
$r = 1+2\cos(2\theta)$

$r = 1+2\cos(2(-\theta))$

$= 1+2\cos(-2\theta)$

$= 1+2\cos(2\theta)$

Yes



$-r = 1+2\cos(2\theta)$

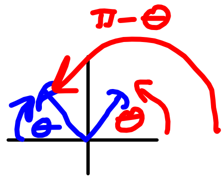
NO

$r = 1+2\cos(2(\theta+\pi))$

$= 1+2\cos(2\theta+2\pi)$

$= 1+2\cos(2\theta)$

Yes



$-r = 1+2\cos(2(-\theta))$

$-r = 1+2\cos(-2\theta)$

$-r = 1+2\cos(2\theta)$

NO

$r = 1+2\cos(2(\pi-\theta))$

$= 1+2\cos(2\pi-2\theta)$

$= 1+2\cos(-2\theta)$

$= 1+2\cos(2\theta)$

Yes.

Linear Operators

$L(ax+by) = aL(x) + bL(y)$, where a, b are real and x & y are functions (or vectors)

$$\frac{d}{dx}\left[3f(x) + 5g(x)\right] = 3\frac{df}{dx} + 5\frac{dg}{dx}$$

$$\frac{d}{dx}(af + bg) = a\frac{df}{dx} + b\frac{dg}{dx}$$

$$\int (af + gy) = a\int f + b\int g$$

$$\int (3\sin x + 5\cos x)\,dx$$

$$= 3\int \sin x\,dx + 5\int \cos x\,dx$$

§11.2 Series

Sequence $a_1, a_2, a_3, \ldots, a_n, a_{n+1}, \ldots$

Series $a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n + a_{n+1} + \cdots$

$$= \sum_{k=1}^{\infty} a_k$$

$$\int_1^{\infty} \frac{dx}{x^2} \qquad \sum_{k=1}^{\infty} \frac{1}{k^2}$$

Integral Test for

$$S = \sum_{k=1}^{\infty}$$

$$S_n = \sum_{k=1}^{n}$$ Then look at the sequence $\{S_n\}_{n=1}^{\infty}$

Geometric Sum

$$S = a + ar + ar^2 + ar^3 + ar^4 + \cdots$$

$$S_n = a + ar + \cdots + ar^{n-1}$$

$$-rS_n = \quad ar + \cdots + ar^{n-1} + ar^n$$

$$S_n - rS_n = a - ar^n$$

$$S_n(1-r) = a - ar^n$$

$$S_n = \frac{a - ar^n}{1-r} = a\left(\frac{1-r^n}{1-r}\right) \xrightarrow{n \to \infty} a\left(\frac{1}{1-r}\right)$$

if $-1 < r < 1$

$$\lim (af \pm bg) = a \lim f + b \lim g$$

a, b constants
f, g functions.

You can always do this, if $\lim f$ & $\lim g$ exist and are real.

$$\lim (fg) = \lim f \lim g$$

$$\lim \left(\frac{f}{g}\right) = \frac{\lim f}{\lim g}, \text{ provided } \lim g \neq 0$$

$$\lim (f^7) = (\lim f)^7$$

Sometimes $\lim f$ & $\lim g$ $\nexists$ but $\lim (f+g)$ converges.

Example of this last:

$$0 = \lim_{n\to\infty} \left(\frac{1}{n}\right) = \lim \left(n + \frac{1}{n} - n\right)$$

$$= \lim \left(n + \frac{1}{n}\right) - \lim (n)$$

$$= \infty - \infty ?!$$