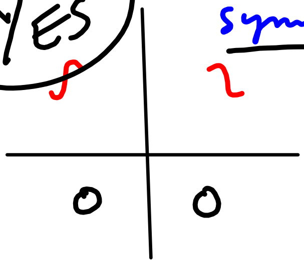
Practice Test 4

YES

#5 $r = 1 + 2\sin\theta$



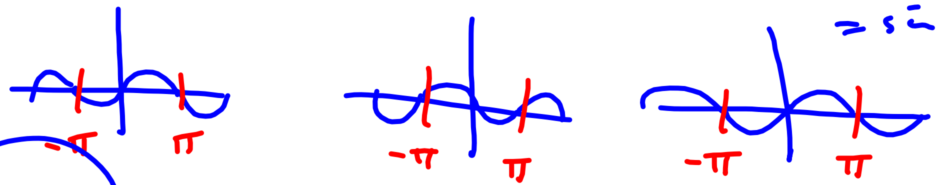
symmetry about y-axis $\Longleftrightarrow \theta = \frac{\pi}{2}$

Replace θ by $\pi - \theta = -(\theta - \pi)$

$r = 1 + 2\sin(-(\theta - \pi))$

$\sin(-(\theta - \pi))$

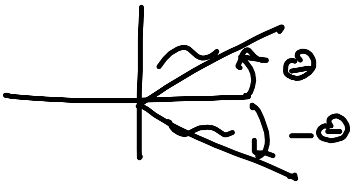
$\sin\theta$ $\quad \sin(-\theta)$ $\quad \sin(-(\theta - \pi)) = \sin\theta$



No

Yes $1 + 2\sin\theta = 1 + 2\sin(\pi - \theta)$

Symmetry about x-axis $\Longleftrightarrow$ polar axis $\Longleftrightarrow \theta = 0$



$r = 1 + 2\sin(\theta)$

$r = 1 + 2\sin(-\theta)$

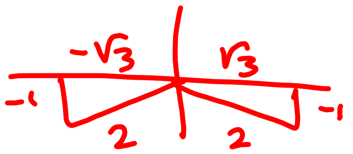
$= 1 - 2\sin\theta$ No.

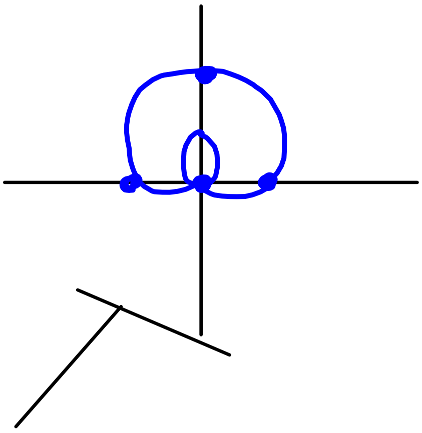
Graph:

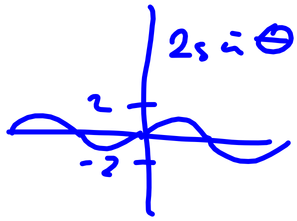
$r = 1 + 2\sin\theta$

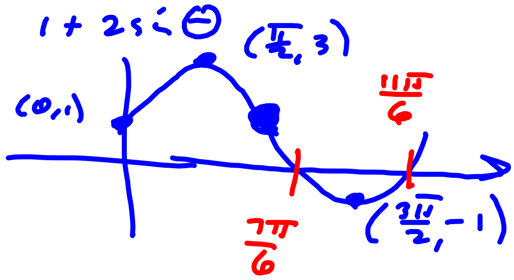
$1 + 2\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$



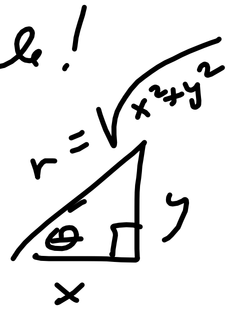






$r = 10\sin\theta$ is a circle!

$r = 10\dfrac{y}{r}$

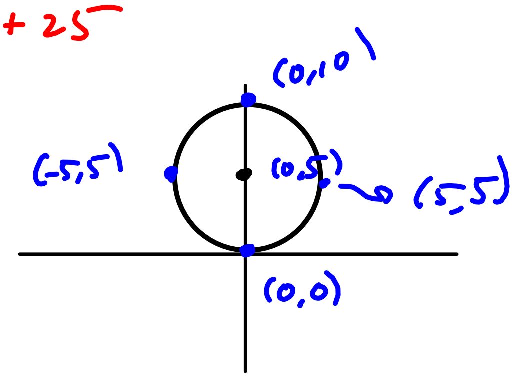


$r^2 = 10y$

$x^2 + y^2 = 10y$

$x^2 + y^2 - 10y + 5^2 = 0 + 25$

$x^2 + (y-5)^2 = 25$



$r = \csc\theta \cot\theta$

$= \frac{r}{y} \cdot \frac{x}{y}$

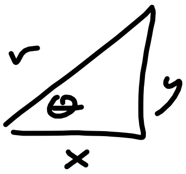
$1 = \frac{x}{y^2}$

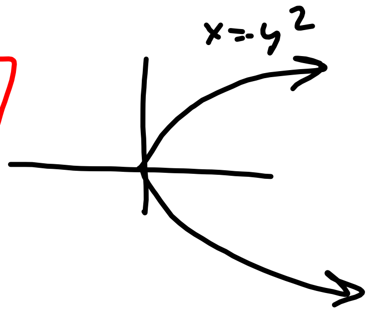
I suck

$\boxed{x = y^2}$

$y = \pm\sqrt{x}$

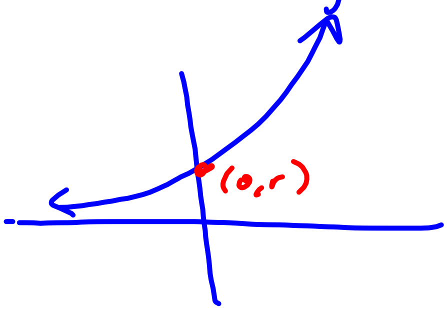
I'm better

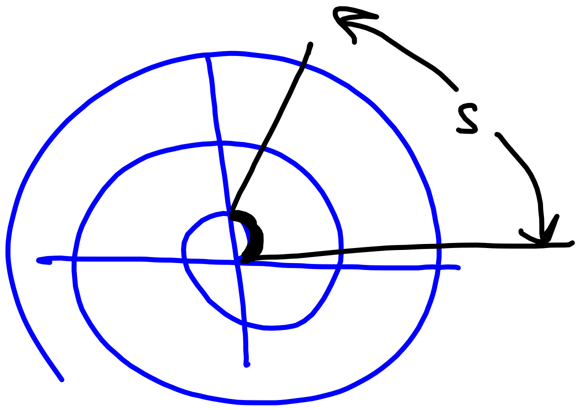




$r = e^{2\theta}$ $\quad 0 \le \theta \le \frac{\pi}{2}$

Arc Length





$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$S = \int_0^{\frac{\pi}{2}} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$r = e^{2\theta}$

$\left(\frac{dr}{d\theta}\right)^2 = \left(2e^{2\theta}\right)^2 = 4e^{4\theta}$

$r^2 = e^{4\theta}$

$$= \int_0^{\frac{\pi}{2}} \sqrt{5e^{4\theta}}\, d\theta$$

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = e^{4\theta} + 4e^{4\theta} \quad * \left(e^{2\theta}\right)^2$$

$$= 5e^{4\theta}$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{5}\, e^{2\theta}\, d\theta = \frac{\sqrt{5}}{2} \int_0^{\frac{\pi}{2}} e^{2\theta} \cdot 2\, d\theta$$

$$= \frac{\sqrt{5}}{2} e^{2\theta} \Big]_0^{\frac{\pi}{2}} = \boxed{\frac{\sqrt{5}}{2} e^{\pi} - \frac{\sqrt{5}}{2}}$$