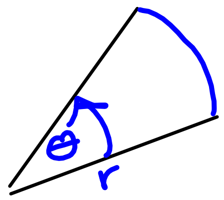
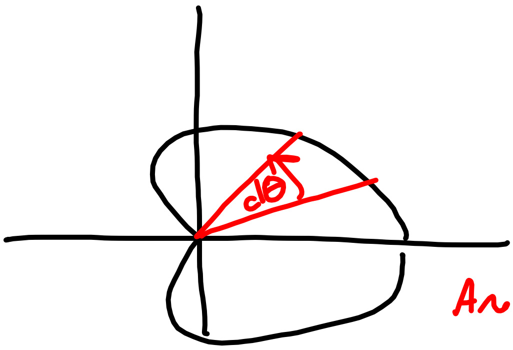
§10.4 areas & arc lengths in polar coordinates.



Area $= \frac{1}{2} r^2 \theta$

Area of circle $= \pi r^2$
corresponds to angle of
$\theta = 2\pi$

$\pi r^2 = \frac{1}{2}(2\pi) r^2$
$= \frac{1}{2}\theta r^2$

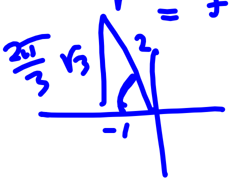


Area of sector when θ = small $= \Delta\theta$
$= d\theta$ is
$\frac{1}{2} r^2 d\theta$

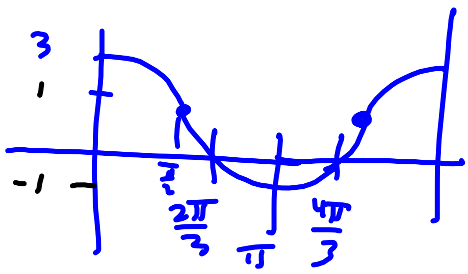
$$\text{Area} = \frac{1}{2}\int_\alpha^\beta r^2 d\theta = \frac{1}{2}\int_\alpha^\beta f(\theta)^2 d\theta$$

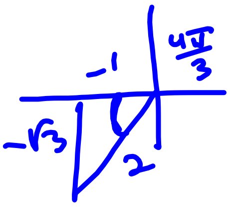
E

$r = f(\theta) = 1 + 2\cos\theta$

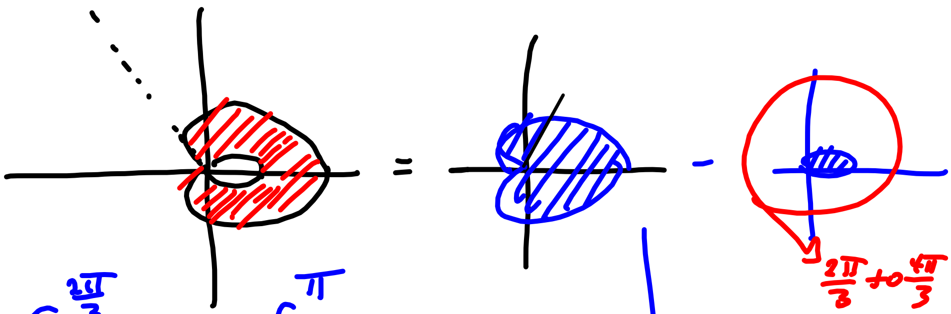


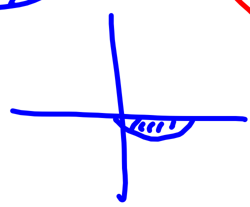
$2\cos\theta = -1$
$\cos\theta = -\frac{1}{2}$





Find area outside the inner loop





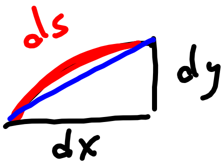
$$= 2\int_0^{\frac{2\pi}{3}} - 2\int_{\frac{2\pi}{3}}^{\pi}$$

$$= 2\cdot\frac{1}{2}\int_0^{\frac{2\pi}{3}}(1+2\cos\theta)^2\,d\theta - 2\cdot\frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi}(1+2\cos\theta)^2\,d\theta$$

$$(2\cos\theta+1)^2 = 4\cos^2\theta + 4\cos\theta + 1$$

$$= 4\left(\frac{1+\cos(2\theta)}{2}\right) + 4\cos\theta + 1$$

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx \quad = \text{arc length increment}$$



Approximate ds w/ straight line segment

$$ds \approx \sqrt{(dx)^2 + (dy)^2}$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\quad dt$$

$$\sqrt{\frac{(dx)^2}{(dx)^2} + \frac{(dy)^2}{(dx)^2}}\quad dx$$

Polars

$$ds = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$x = r\cos\theta$ $y = r\sin\theta$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$$

$(a-b)^2 = a^2 - 2ab + b^2$

$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$$

$$\left(\frac{dx}{d\theta}\right)^2 = \left(\frac{dr}{d\theta}\right)^2\cos^2\theta - 2\frac{dr}{d\theta}\cos\theta\, r\sin\theta + r^2\sin^2\theta$$

$$+ \left(\frac{dy}{d\theta}\right)^2 = \left(\frac{dr}{d\theta}\right)^2\sin^2\theta + 2\frac{dr}{d\theta}\sin\theta\, r\cos\theta + r^2\cos^2\theta$$

$$\left(\frac{dr}{d\theta}\right)^2\cos^2\theta + \left(\frac{dr}{d\theta}\right)^2\sin^2\theta + r^2\sin^2\theta + r^2\cos^2\theta$$

$$= \left(\frac{dr}{d\theta}\right)^2\left(\cos^2\theta + \sin^2\theta\right) + r^2\left(\sin^2\theta + \cos^2\theta\right)$$

$$= \left(\frac{dr}{d\theta}\right)^2 + r^2 \quad !$$

$$ds = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Arclength in polars:

$$\int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

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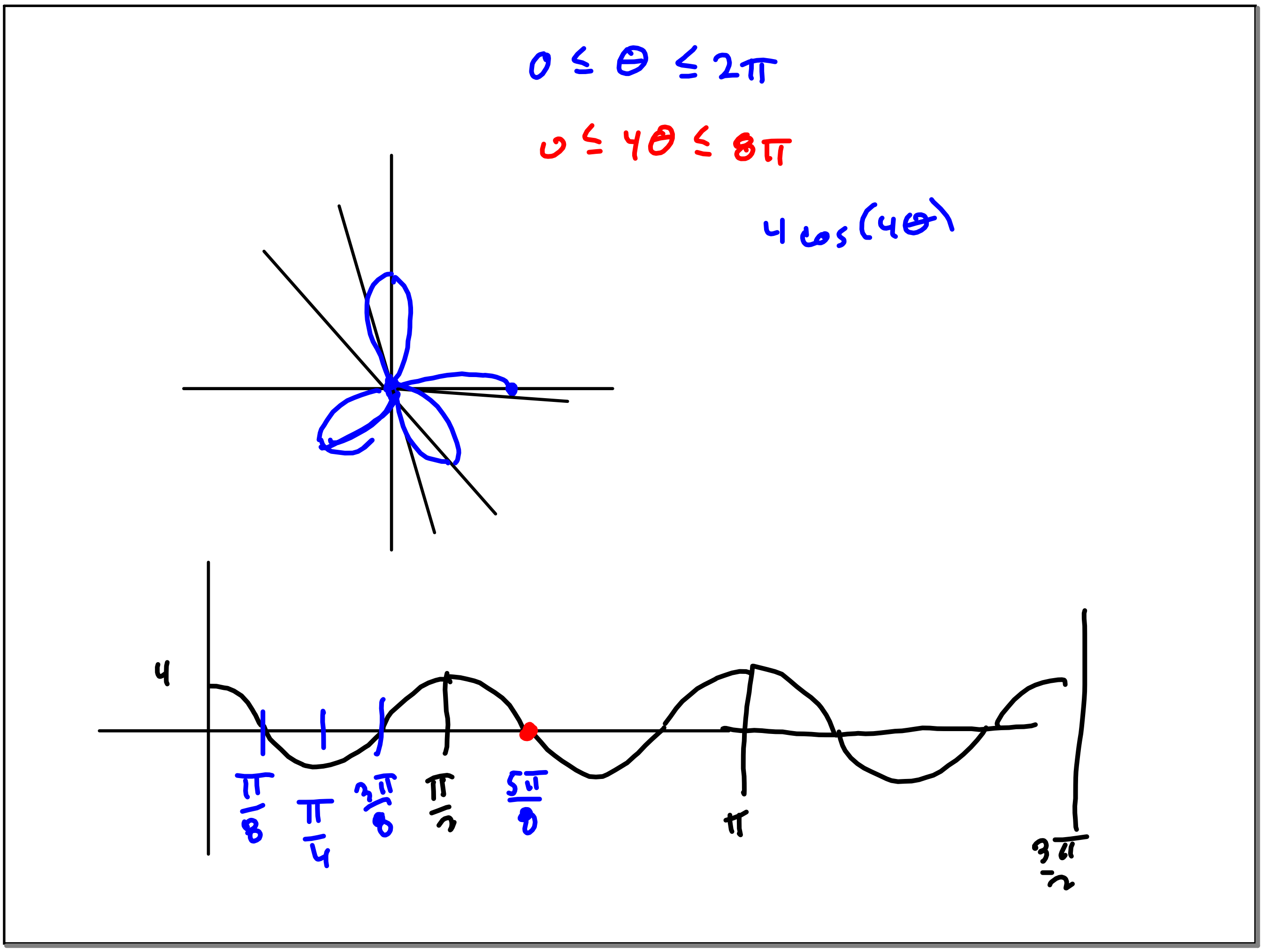
homogeneous heterogeneous

$$y' \sin y = x$$

$$(\sin y)\frac{dy}{dx} = x$$

$$\int \sin y \, dy = \int x \, dx$$

$$\int y' = y$$


0 ≤ θ ≤ 2π
0 ≤ 4θ ≤ 8π
4 cos(4θ)
4
π/8
π/4
3π/8
π/2
5π/8
π
3π/2