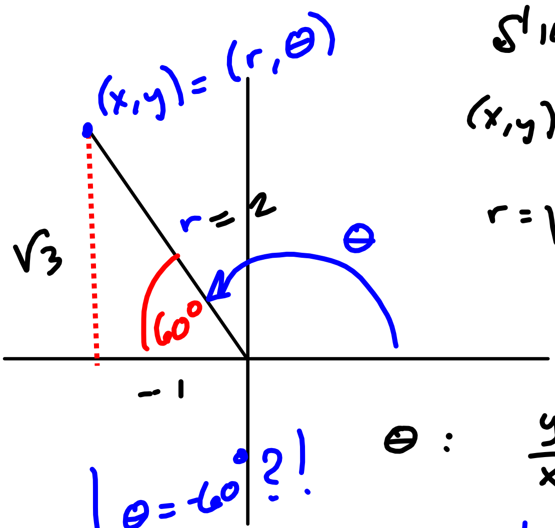
§10.3 Polar coordinates



$(x,y) = (-1, \sqrt{3})$

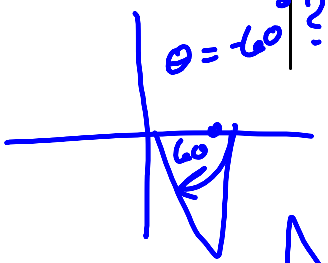
$r = \sqrt{x^2+y^2} = \sqrt{1^2+\sqrt{3}^2}$

$= \sqrt{1+3} = \sqrt{4} = 2$

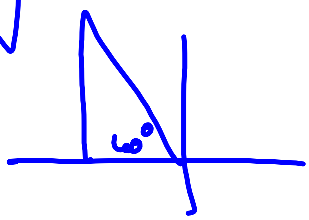
$\theta : \quad \frac{y}{x} = \tan\theta = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

So, $\theta = \tan^{-1}(-\sqrt{3}) = \arctan(-\sqrt{3})$

$= -60°$, right? Wrong.



$180° - 60° = 120° = \frac{2\pi}{3}$

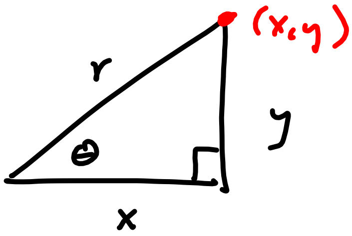


$R(\text{arctangent}) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = (-90°, 90°)$

$r^2 = x^2 + y^2$

$\frac{x}{r} = \cos\theta$

$x = r\cos\theta \;,\; y = r\sin\theta$



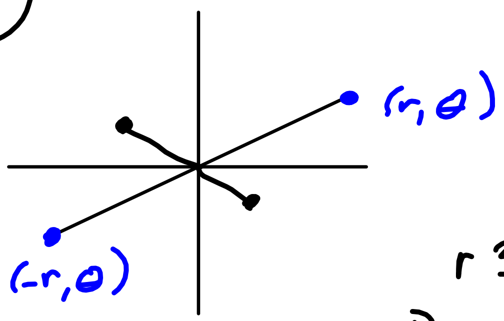
$r = f(\theta) = 1 + \cos\theta$

$r^2 = 1 + 2\cos\theta + \cos^2\theta$

Want to do calculus with this & graph it!

Symmetry:

1



Suppose $(-r,\theta)$ is also on the graph.

Symmetry thru the origin (Pole)

$r^2 = 1+\cos\theta$

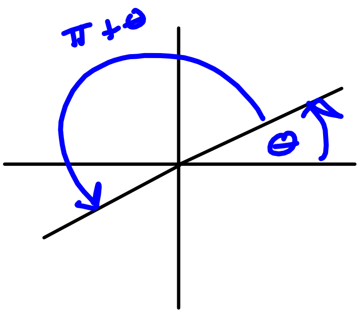
Replacing r by $-r$ gives an equivalent equation.

$(-r)^2 = 1+\cos\theta$

$r^2 = 1+\cos\theta$

Note also that

θ & $\pi+\theta$ lie on the same line.



This is also symmetry thru the pole.

$r \longleftrightarrow -r$

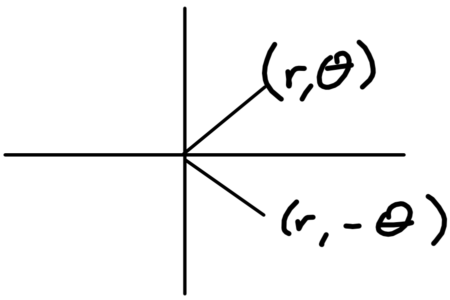
$\theta \longleftrightarrow \theta+\pi$

$r = \tan\theta$

$r = \tan(\theta+\pi) = \tan\theta$

Sometimes you have symmetry thru the pole, but fails $\theta+\pi$ test.

(2) Note that (r,θ) & $(r,-\theta)$ are reflections thru the x-axis
polar axis.



If you replace θ by $-\theta$ & obtain equivalent eq'n, then symmetric about the polar axis.

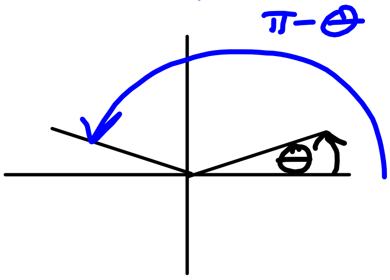
$r = 1+\cos\theta$

$r = 1+\cos(-\theta)$

$r = 1+\cos\theta$

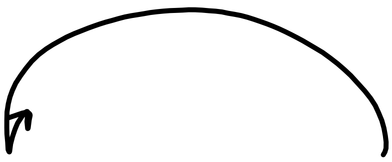
equivalent.

③ Symmetry about the line $\theta = \frac{\pi}{2}$

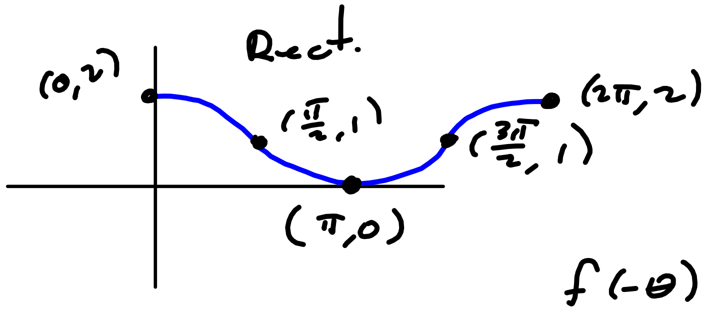


If replacing θ by $\pi - \theta$ gives equivalent equation, then symmetry about $\theta = \frac{\pi}{2}$

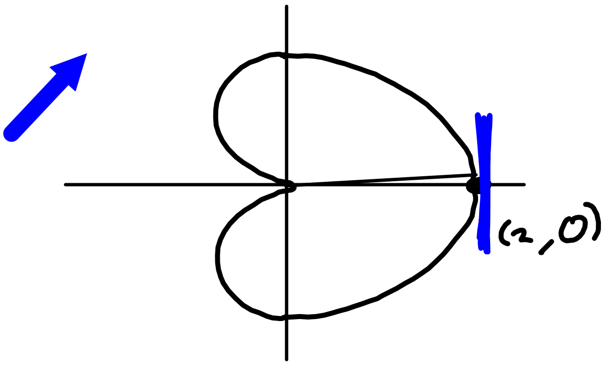
($x = 0$, y-axis).



$r = f(\theta) = 1 + \cos\theta$



$f(-\theta) = 1 + \cos(-\theta)$
$= 1 + \cos\theta$

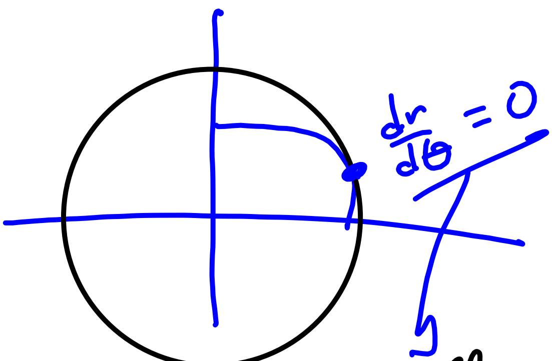


Points of interest:

$\frac{dy}{dx} = 0$ $\left(\frac{dy}{d\theta} = 0\right)$

$\frac{dx}{d\theta} = 0$

$\frac{dr}{d\theta}$: where (r, θ) is a point where the graph would be parallel to a circle of radius r.



will give max (local) r-value.

$r = f(\theta)$

$$x = x(\theta) = r\cos\theta$$
$$y = y(\theta) = r\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{d}{d\theta}[r\sin\theta]}{\frac{d}{d\theta}[r\cos\theta]}$$

& r is $f(\theta)$

$$= \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$$x = r\cos\theta$$
$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta$$

$$y = r\sin\theta$$
$$\frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$$

$\frac{dy}{d\theta} = 0$ means $\longleftrightarrow$

$\frac{dx}{d\theta} = 0$ means $\updownarrow$

When Both are zero, we need L'HOPITAL

$$\frac{dy}{dx} = \frac{0}{0}$$