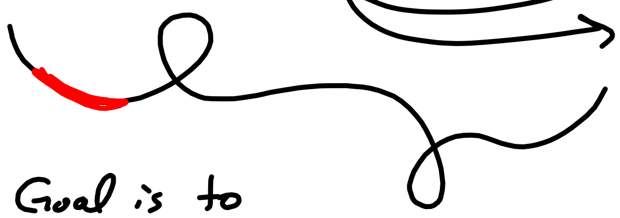
§10.1

$x = f(t)$, $y = g(t)$

$(x, y) = (f(t), g(t))$ are points in the plane

Assume $y = F(x)$ locally $\rightarrow y = x(t)$



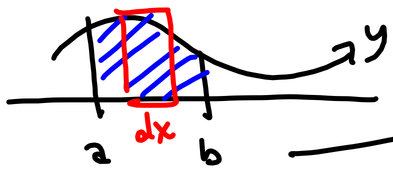
Goal is to Find $\frac{dy}{dx}$

$$\frac{dy}{dt} = \cancel{\frac{d[F(x)]}{dt} = \frac{d[F(x(t))]}{dt}}$$

$$\frac{dy}{dt} \qquad \frac{d(y(x(t)))}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Areas:



$$\int_{a=x}^{b=x} y\, dx$$

$$\boxed{\begin{array}{l} y = g(t) \\ x = f(t) \end{array}}$$

$$\frac{dx}{dt} = f'(t)$$

$$= \int_{a=x}^{b=x} y\, f'(t)\,dt$$

$dx = f'(t)\, dt$

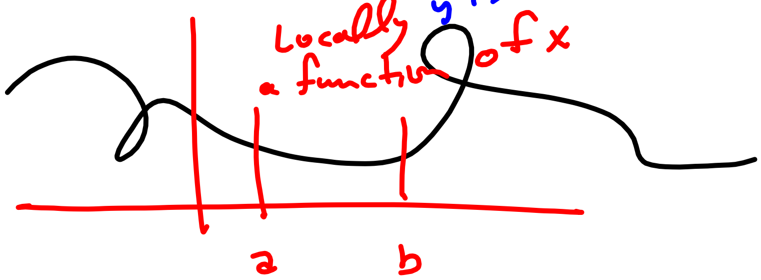
$x = f(t) = b$
solve for t to get β

$x = f(t) = a$
Solve for t to get α

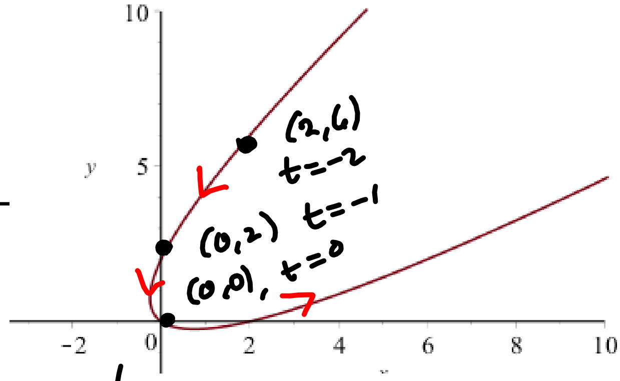
$$= \int_{\alpha=t}^{\beta=t} y\, f'(t)\,dt = \int_{\alpha}^{\beta} g(t) f'(t)\,dt$$

This only makes sense if

$y = F(x)$ locally



$x = t^2 + t \quad , \; y = t^2 - t$



Eliminating the parameter

$t^2 + t = x$

$t^2 + t$

$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$t^2 + t + \left(\frac{1}{2}\right)^2 = x + \frac{1}{4}$

$\left(t + \frac{1}{2}\right)^2 = x + \frac{1}{4}$

$t + \frac{1}{2} = \pm\sqrt{x + \frac{1}{4}}$

$t = -\frac{1}{2} \pm \sqrt{x + \frac{1}{4}}$

$= -\frac{1}{2} \pm \frac{1}{2}\sqrt{4x+1}$

$\Rightarrow y = t^2 - t$

$y = \left(-\frac{1}{2} \pm \frac{1}{2}\sqrt{4x+1}\right)^2 - \left(-\frac{1}{2} \pm \frac{1}{2}\sqrt{4x+1}\right)$

$= \frac{1}{4} \pm (2)\left(-\frac{1}{2}\right)\left(\frac{1}{2}\sqrt{4x+1}\right) + \frac{1}{4}(4x+1)$

$+ \frac{1}{2} \mp \frac{1}{2}\sqrt{4x+1}$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$= \frac{1}{4} \mp \frac{1}{2}\sqrt{4x+1} + x + \frac{1}{4} + \frac{1}{2} \mp \frac{1}{2}\sqrt{4x+1}$

$= 1 \mp \sqrt{4x+1} + x$

Top $\frac{1}{2}$ & Bottom half, w/ domain

$= \left[-\frac{1}{4}, \infty\right)$

Recall the arc length increment
(differential of arc length.)

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Now, $x = f(t) = x(t)$
$y = g(t) = y(t)$

$x = f(t)$

$dx = f'(t)\,dt = \frac{dx}{dt}\cdot dt$

$3 = \sqrt{3^2}$

$\frac{dx}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2}$

provided $\frac{dx}{dt} \geq 0$

$$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,\frac{dx}{dt}\,dt$$

$$= \sqrt{\left(1+\left(\frac{dy}{dx}\right)^2\right)\left(\frac{dx}{dt}\right)^2}\,dt$$

$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \frac{\left(\frac{dy}{dt}\right)^2}{\cancel{\left(\frac{dx}{dt}\right)^2}}\cancel{\left(\frac{dx}{dt}\right)^2}}\,dt$$

$$= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Arc Length for parametrics:

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Surface area $x = f(t)$, $y = g(t)$

x-axis:

$$A = 2\pi \int_a^b y \, ds$$

$$= 2\pi \int_\alpha^\beta y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$$

$$= 2\pi \int_\alpha^\beta g(t) \sqrt{f'(t)^2 + g'(t)^2} \, dt$$