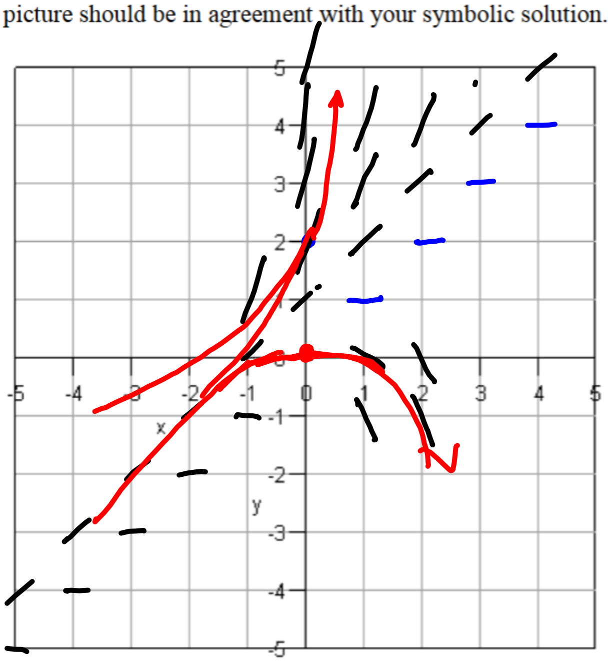
4. Sketch the direction field for $y' = y - x$, using the graph paper provided. Show the graph of the solutions corresponding to $y(0) = 2$ and $y(0) = 0$ on your direction field. Then solve this linear differential equation for y (Hint: Re-write in the form $y' - y = -x$.) Your picture should be in agreement with your symbolic solution.



5. Use the characteristic polynomial to find the general solution to $y'' - 16y = 0$. Then find the particular solution corresponding to the boundary conditions $y'(0) = y(0) = 1$.

Linear 2nd-order ordinary differential equation with constant coefficients.

$$y'' - 16y = 0$$
$$(D^2 - 16)y = 0$$
$$\boxed{D^2 - 16 = 0}$$

Characteristic Polynomial.

$$D = \pm 4 \Longrightarrow c_1 e^{4t} + c_2 e^{-4t} = y$$

$$y'' + 16y = 0$$
$$\vdots$$
$$\boxed{y = c_1 e^{4it} + c_2 e^{-4it}} = A\cos(4t) + B\sin(4t)$$

$$\sin t = \frac{e^{it} - e^{-it}}{2i}$$

$$\cos t = \frac{e^{it} + e^{-it}}{2}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 2x_2 \\ 3x_1 + 4x_2 \end{bmatrix}$$

$2 \times 2 \quad 2 \times 1 \quad 2 \times 1$

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\bar{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$,

and $B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$

$$x_1 + 2x_2 = 3$$
$$3x_1 + 4x_2 = 7$$

Then the system can be distilled into the MATRIX Equation $A\bar{X} = B$

$$\Rightarrow \bar{X} = \frac{B}{A}$$

A^{-1} satisfies $AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I =$ "1"

for matrices. $I\bar{X} = \bar{X}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 7 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$$

Eigenvalues a real # λ such that

$A\underline{X} = \lambda \underline{X}$

A special vector called eigenvector.

$A\underline{X} = \lambda \underline{X}$ $\qquad \lambda \underline{X} = \lambda I \underline{X}$

$A\underline{X} - \lambda \underline{X} = 0$

$A\underline{X} - \lambda I \underline{X} = 0$

$(A - \lambda I)\underline{X} = 0$

The determinant of this matrix is the characteristic polynomial of the matrix.

$|A - \lambda I| = 0$ solutions give eigenvalues.

$$-\frac{1}{2}\int_0^3 \frac{-2x\,dx}{\sqrt{25-x^2}}$$

$u = 25 - x^2$

$du = -2x\,dx$

$$-\frac{1}{2}\int_{0=x}^{3=x} u^{-\frac{1}{2}}\,du$$

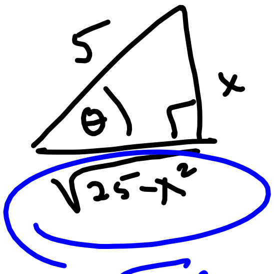
u^{-}

$$\int u^{-\frac{1}{2}}\,du = 2u^{\frac{1}{2}} + C, \text{ so}$$

$$-\frac{1}{2}\left[2\sqrt{25-x^2}\right]_0^3$$

$$= -\frac{1}{2}\left[2u^{\frac{1}{2}}\right]_{0=x}^{3=x}$$

$$= -\left[\sqrt{25-x^2}\right]_0^3$$



So $\frac{\sqrt{25-x^2}}{5} = \cos\theta$

$\frac{x}{5} = \sin\theta$

$x = 5\sin\theta$

$dx = 5\cos\theta\, d\theta$

$= -[4-5] = 1$

$\sqrt{25-x^2} = \sqrt{25-25\sin^2\theta}$

$= \sqrt{25(1-\sin^2\theta)}$

$= 5\sqrt{1-\sin^2\theta}$

$= 5\sqrt{\cos^2\theta}$

$= 5|\cos\theta| = 5\cos\theta$

$x=0 = 5\sin\theta$

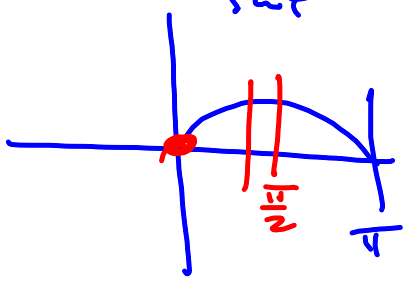
$\sin\theta = 0$

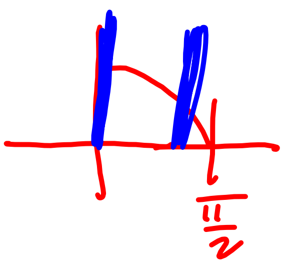
$\theta = 0$

$x=3 = 5\sin\theta$

$\sin\theta = \frac{3}{5}$

$\theta = \arcsin\frac{3}{5}$



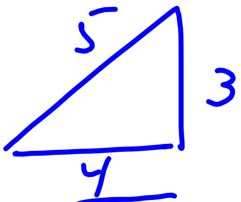


$$\Rightarrow \int_0^{\arcsin(3/5)} \frac{5\sin\theta \cdot 5\cos\theta \, d\theta}{5\cos\theta}$$

$$= 5\int_0^{\arcsin\left(\frac{3}{5}\right)} \sin\theta \, d\theta = 5\left[-\cos\theta\right]_0^{\arcsin\left(\frac{3}{5}\right)}$$

$$= -5\left[\cos\left(\arcsin\left(\frac{3}{5}\right)\right) - 1\right]$$

$= 5$



$$\sqrt{25-9} = \sqrt{16} = 4$$

$$= -5\left[\frac{4}{5} - 1\right] = -5\left[-\frac{1}{5}\right] = 1$$