$y' + x\sqrt{y} = x^2$ nonlinear, because $\sqrt{y}$

$y' + P(x)y = Q(x)$

$2xy' + y = 2\sqrt{x}$ is linear, after some manipulation.

$$y' + \frac{1}{2x}y = \frac{2\sqrt{x}}{2x} = \frac{1}{\sqrt{x}}$$

$$\frac{x^{\frac{1}{2}}}{x^1} = \frac{1}{x^{1-\frac{1}{2}}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$$

$$y' + \frac{1}{2x}y = \frac{1}{\sqrt{x}} \iff y' + P(x)y = Q(x)$$

$$I = e^{\int P(x)\,dx} = e^{\int \frac{1}{2x}dx} = e^{\frac{1}{2}\int\frac{dx}{x}} = e^{\frac{1}{2}\ln|x|} = e^{\frac{1}{2}\ln x}$$

$$= \sqrt{x}$$

why? $\sqrt{x}$ in original problem $\Rightarrow x \geq 0$ for it to be real.

Props of logs
$$\left\{\begin{array}{l} \frac{1}{2}\ln x = \ln\left(x^{\frac{1}{2}}\right) \Rightarrow e^{\ln(x^{\frac{1}{2}})} = x^{\frac{1}{2}} = \sqrt{x} \\ e^{\frac{1}{2}\ln x} = \left(e^{\frac{1}{2}}\right)^{\ln x} = \left(e^{\ln x}\right)^{\frac{1}{2}} = x^{\frac{1}{2}} = \sqrt{x} \end{array}\right.$$

$$\left(a^b\right)^c = a^{bc}$$

$\Rightarrow I(x)\,[\text{eq'n}]$ is

eq'n
$$\boxed{y' + \frac{1}{2x}y = \frac{1}{\sqrt{x}}}$$

$$\underbrace{\sqrt{x}\left(y' + \frac{1}{2x}y\right)}_{\frac{d}{dx}[I(x)y]} = \sqrt{x}\left(\frac{1}{\sqrt{x}}\right) = 1$$

$$\frac{d}{dx}[I(x)y] = I'(x)y + I(x)y'$$

$$\frac{d(\sqrt{x}\,y)}{dx} = \sqrt{x}\,y' + \frac{1}{2\sqrt{x}}y = 1$$

$$\frac{1}{2\sqrt{x}}y + \sqrt{x}\,y' = \frac{d}{dx}\left[\sqrt{x}\,y\right] = \frac{1}{2}x^{-\frac{1}{2}}y + \sqrt{x}\,y'$$

$$\frac{d(\sqrt{x}\,y)}{dx} = 1$$

$$\int d(\sqrt{x}\,y) = \int dx$$

$$\sqrt{x}\,y = 1 + C$$

$$\int \frac{d(\sqrt{x}\,y)}{dx}\,dx = \int 1\,dx$$

$$\sqrt{x}\,y = x + C$$

$$y = \frac{x}{\sqrt{x}} + \frac{C}{\sqrt{x}} = \frac{1}{\sqrt{x}} + \frac{C}{\sqrt{x}} = \boxed{\frac{C+1}{\sqrt{x}} = y}$$

for any $C \in \mathbb{R}$

$y = 2\ln\left(\sin\left(\frac{x}{2}\right)\right)$ $\qquad \frac{\pi}{3} \le x \le \pi$

Arc length.

$y' = 2\left(\frac{\frac{1}{2}\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}\right) = \tan\left(\frac{x}{2}\right)$

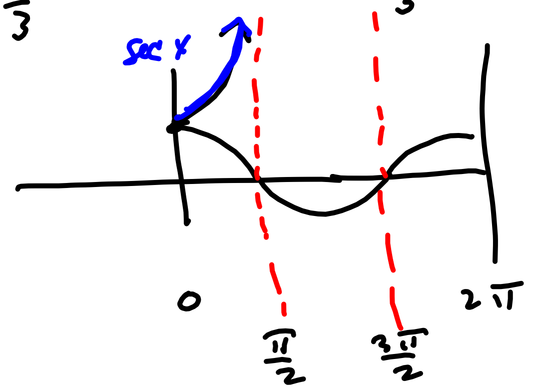
GOOD TEST QUESTION

$\int_{\frac{\pi}{4}=x}^{\frac{3\pi}{4}=x}$

$\Rightarrow (y')^2 = \tan^2\left(\frac{x}{2}\right) \Rightarrow$

$$\int_{\frac{\pi}{3}}^{\pi} \sqrt{1+y'^2}\,dx = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \sqrt{1+\tan^2\left(\frac{x}{2}\right)}\,dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}}$$

$$= \int_{\frac{\pi}{3}}^{\pi} \sqrt{\sec^2\left(\frac{x}{2}\right)}\,dx = \int_{\frac{\pi}{3}}^{\pi} \left|\sec\left(\frac{x}{2}\right)\right|\,dx$$



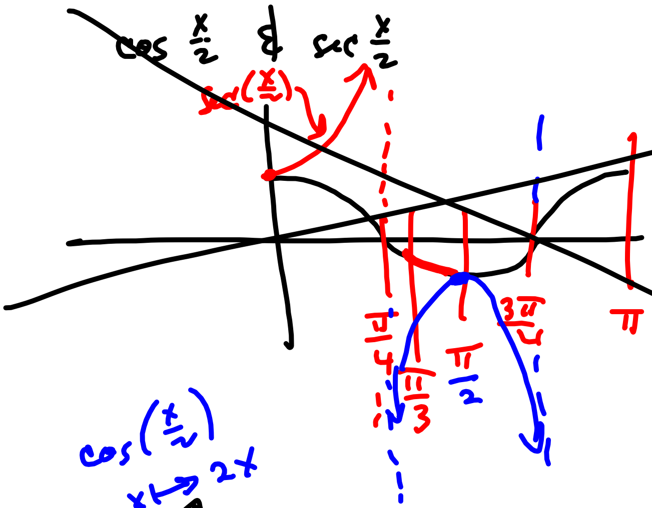
$\frac{\pi}{3} < x < \pi$

$\frac{\pi}{6} < \frac{x}{2} < \frac{\pi}{2}$

$\sec(\frac{x}{2})$ isn't cnt^{s} on $[\frac{\pi}{6}, \frac{\pi}{2}]$

It blows up @ $x = \frac{\pi}{2}$

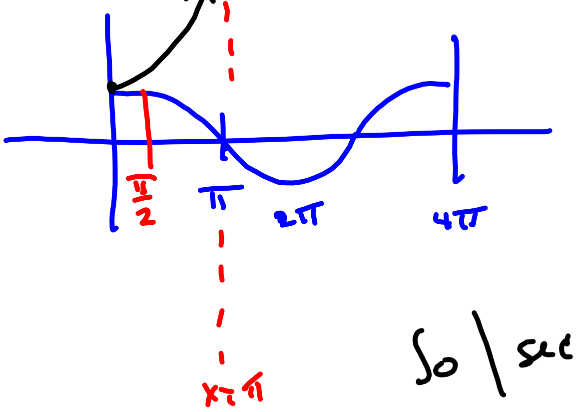
2



This is $\cos(2x)$ NOT $\cos(\frac{x}{2})$

$f(x) \longmapsto f(\frac{x}{2})$

$(x,y) \longmapsto (2x,y)$



So $|\sec(\frac{x}{2})| = \sec(\frac{x}{2})$

$$\int_{\frac{\pi}{3}=x}^{\pi=x} \sec\left(\frac{x}{2}\right)dx$$

$u=\frac{x}{2}$

$x=2u$

$dx=2\,du$

$$=2\int_{u=\frac{\pi}{6}}^{u=\frac{\pi}{2}} \sec u\,du$$

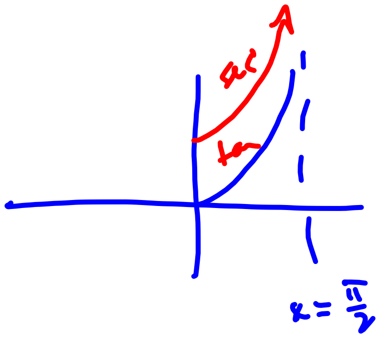
Improper integral.

$$2\int_{\frac{\pi}{6}}^{t} \sec u\,du = 2\ln\left|\sec u+\tan u\right|\Big]_{0}^{t}$$

$$2\ln|\sec t+\tan t| - 2\ln|\sec 0+\tan 0|$$

$$\text{"}\underline{t\to\frac{\pi}{2}^{-}}\text{"}\ 2\ln\left|\sec\frac{\pi}{2}+\tan\frac{\pi}{2}\right| - 2\ln 1$$

$$=2\ln\left|\underset{\pm\infty}{\sec\frac{\pi}{2}}+\underset{\pm\infty}{\tan\frac{\pi}{2}}\right|$$



$\nexists$

Teacher sucked.

$\boxed{\text{about x-axis} \quad 2\pi \int y \, ds}$

$\boxed{\text{about y-axis} \quad 2\pi \int x \, ds}$

$x = g(y)$

$g'(y)^2$

$y = f(x)$:

$2\pi \int f(x) \, ds$

$ds \to \sqrt{1 + \frac{dy}{dx}^2} \, dx$

$x = g(y)$

$2\pi \int y \, ds$

$ds \to \sqrt{1 + \frac{dx}{dy}^2} \, dy$

$y = f(x)$

$2\pi \int x \, ds$

$x = g(y)$

$2\pi \int g(y) \, ds$