§9.3 #15

$$x \ln x = y\left(1+\sqrt{3+y^2}\right)y'$$

$$y(1)=1$$

Constant is arbitrary.

$$\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 = \frac{1}{2}y^2 + \frac{1}{3}(3+y^2)^{\frac{3}{2}} + \tilde{C}$$

$$\hat{C} - C_2 = C$$

$$\int y + \frac{1}{2}\int\left(\sqrt{3+y^2}\right)(2y\,dy)$$

$u = 3+y^2$

$du = 2y\,dy$

$$\frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}} + C$$

$$= \frac{1}{3}(3+y^2)^{\frac{3}{2}} + C$$

#8 §9.3

$$\frac{dh}{dr} = \frac{rh^2\sqrt{1+r^2}}{\ln h}$$

$$A = \int \frac{\ln h}{h^2}\,dh = \int r\sqrt{1+r^2}\,dr = B$$

$\cancel{u = \ln h}$

$\cancel{du = \frac{1}{h}dh}$

$\cancel{\frac{d}{dx}\ln x = \frac{1}{x}}$

$\cancel{\frac{1}{h}\cdot \ln h}$

$$\int \frac{1}{h^2}\ln h\,dh$$

$u = \ln h \qquad \frac{1}{h^2} = dv = h^{-2}$

$du = \frac{1}{h}dh \qquad v = -h^{-1}$

$$uv - \int v\,du = -\frac{\ln h}{h} - \int (-h^{-1})\left(\frac{1}{h}\right)dh$$

$$= -\frac{\ln h}{h} + \int h^{-2}\,dh$$

$$= -\frac{\ln h}{h} - \frac{1}{h} + C = A$$

$$B = \frac{1}{2}\int \sqrt{1+r^2}\cdot 2r\,dr = \frac{1}{2}\int (1+r^2)^{\frac{1}{2}}\cdot 2r\,dr$$

$$= \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}$$

$$= \frac{1}{3}u^{\frac{3}{2}} = \frac{1}{3}(1+r^2)^{\frac{3}{2}}$$

$A = B$:

$$C + -\frac{\ln h}{h} - \frac{1}{h} = \frac{1}{3}(1+r^2)^{\frac{3}{2}}$$

Looks Tough to solve for h.

$$C + \frac{-\ln h - 1}{h} = \frac{1}{3}(r^2+1)^{\frac{3}{2}}$$