§9.5

Goal: Create a separable equation & integrate to a solution

$$\underbrace{y' + P(x)\,y}_{\text{want this to be (something)}'} = Q(x)$$

$$\int (\text{something})'\,dx = \text{something}$$

$$I(x)\left(y' + P(x)y\right) = \left(I(x)y\right)'$$

$$I(x)y' + I(x)P(x)y = \left(I(x)y\right)'$$

$$= I'(x)y + I(x)y'$$

$$I'(x)y = I(x)P(x)y$$

$$I'(x) = I(x)P(x)$$

$$\frac{I'(x)}{I(x)} = P(x)$$

$$\frac{\frac{dI(x)}{dx}}{I(x)} = P(x)$$

$$\int \frac{du}{u} = \int \frac{dI(x)}{I(x)} = \int P(x)\,dx$$

$$\ln|I(x)| = \int p(x)\,dx$$

Assume $I(x) \geq 0$

$$e^{\ln(I(x))} = e^{\int p(x)\,dx}$$

$I(x) = e^{\int P(x)\,dx}$ is the integrating factor.

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$2xy' + y = 2\sqrt{x}$ $\rightarrow x \geq 0$

$y' + P(x)y = Q(x)$

$$\boxed{y' + \frac{1}{2x}y = \frac{1}{2x}\cdot 2x^{\frac{1}{2}} = x^{-\frac{1}{2}}}$$

$$p(x) = \frac{1}{2x}$$

$+C$

Pick the antiderivative where $C=0$.

$$I(x) = e^{\int P(x)dx} = e^{\frac{1}{2}\int x^{-1}dx} = e^{\frac{1}{2}\ln|x| + C\,?} = e^{\frac{1}{2}\ln(x)}$$

$$= \left(e^{\ln(x)}\right)^{\frac{1}{2}} = x^{\frac{1}{2}} = I(x)$$

$$I(x)\left[y' + \frac{1}{2x}y = x^{-\frac{1}{2}}\right]$$

$$e^{\frac{1}{2}\ln x + C} = e^{\frac{1}{2}\ln x}e^{C} = \hat{C}e^{\frac{1}{2}\ln x}$$

$$x^{\frac{1}{2}}y' + \frac{x^{\frac{1}{2}}}{2x}y = x^{\frac{1}{2}}x^{-\frac{1}{2}} = 1$$

$$\boxed{x^{\frac{1}{2}}y' + \frac{1}{2x^{\frac{1}{2}}}y} = 1$$

$$x^{2b} = (x^2)^b$$

$$(I(x)y)' = \left(x^{\frac{1}{2}}y\right)' = \frac{1}{2}x^{-\frac{1}{2}}y + x^{\frac{1}{2}}y'$$

Is the derivative of $I(x)y$!

$$(I(x)y)' = 1$$

$$\frac{d}{dx}\left[I(x)y\right] = 1$$

$$\int d\left[I(x)y\right] = \int dx$$

$$I(x)y = x + C$$

$$y = \frac{x+C}{I(x)} = \frac{x+C}{x^{\frac{1}{2}}} = \boxed{x^{\frac{1}{2}} + Cx^{-\frac{1}{2}} = y}$$

$$y' = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}Cx^{-\frac{3}{2}}$$

$$y' + \frac{1}{2x}y =$$

$$\frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}Cx^{-\frac{3}{2}} + \frac{1}{2x}\left[x^{\frac{1}{2}} + Cx^{-\frac{1}{2}}\right]$$

$$= \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}Cx^{-\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}} \ \frac{C}{2}x^{-\frac{3}{2}}$$

$$= x^{-\frac{1}{2}}$$

$y' + P(x)y = Q(x)$

$\Longrightarrow I(x) = e^{\int P(x)dx}$ is your integrating factor.

§ 9.5 #s 10, 1, 4, 7

Will do a Test 3 video & sample test next week (early)