

Solution to the logistic equation:

<https://www.youtube.com/watch?v=f2r3QtBIMtk>

### Logistic Population Growth

$P = \text{Pop.}$

$t = \text{time}$

$M = \text{Carrying Capacity}$

$k = \text{"a rate"}$

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

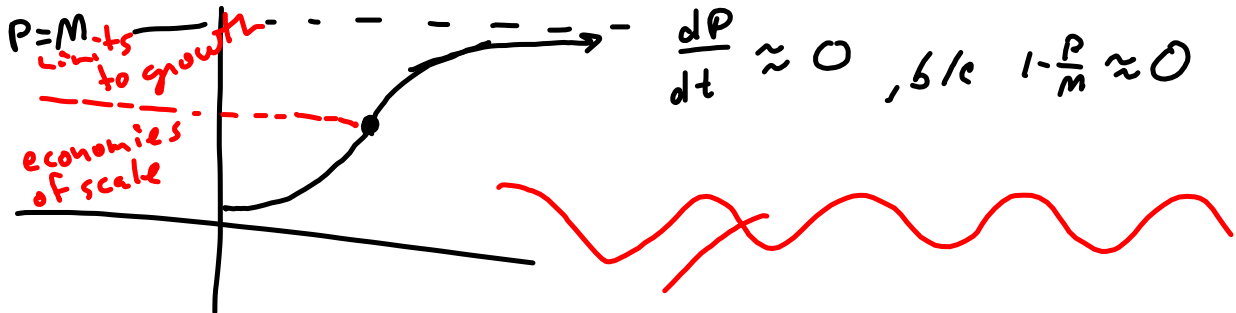
Obeys law of uninhibited growth when  $P \approx \text{SMALL}$ :

Solutions are an S curve

$$\frac{dP}{dt} \approx kP$$

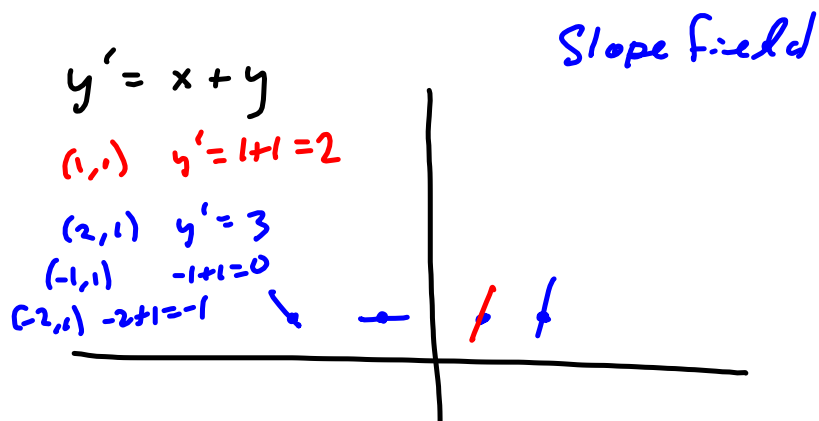
$$P \approx M \rightarrow$$

$$\frac{dP}{dt} \approx 0, \text{ b/c } 1 - \frac{P}{M} \approx 0$$



Solutions look like

$$\frac{k}{1 + ce^{-rt}}$$



5.9.1 Equilibrium Solutions

$$y' = ky \left(1 - \frac{y}{M}\right)$$

$$y' = 0$$

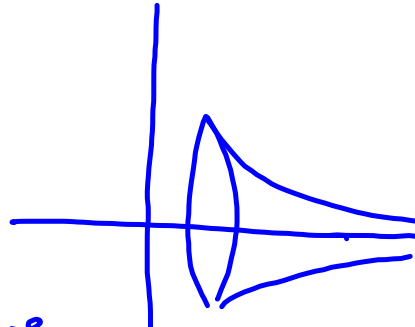
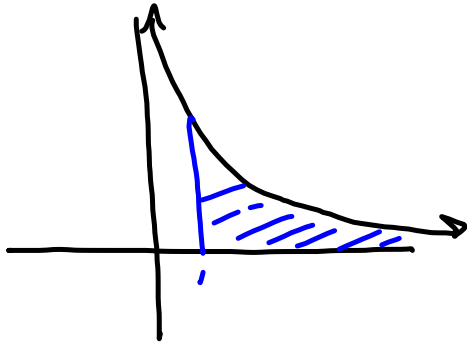
$$y = 0$$

$$y = M$$

GLD Book

§ 8.2 #25  $y = x, 0 \leq y \leq 1$ , about  $y$ -axis

$R = \{(x, y) \mid x \geq 1, 0 \leq y \leq \frac{1}{x}\}$  about  $x$ -axis



$$y = \frac{1}{x}$$

$$y' = -\frac{1}{x^2}$$

$$(y')^2 = \frac{1}{x^4}$$

$$2\pi \int_1^{\infty} y \sqrt{1 + \frac{1}{x^4}} dx$$

$$= 2\pi \int_1^{\infty} \frac{1}{x} \sqrt{1 + \frac{1}{x^4}} dx$$

$$\frac{1}{x} \sqrt{\frac{x^4 + 1}{x^4}}$$

$$= \frac{1}{x^3} \sqrt{x^4 + 1}$$

$\approx \sqrt{x^4} = x^2$

Acting like

$$\frac{1}{x^3} \cdot x^2 \text{ as } x \rightarrow \text{BIG}$$

$$= \frac{1}{x} \text{ and } \int_1^{\infty} \frac{1}{x} dx = \infty$$