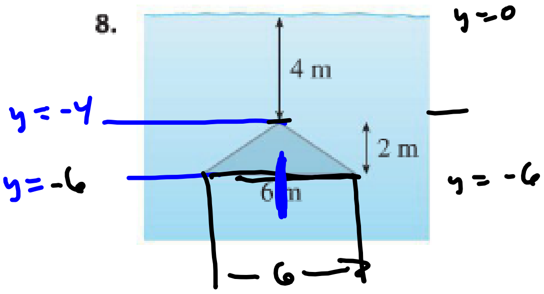
§8.3 #8

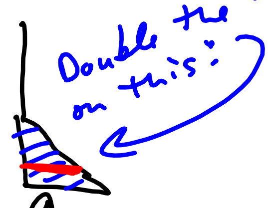
3–11 A vertical plate is submerged (or partially submerged) in water and has the indicated shape. Explain how to approximate the hydrostatic force against one side of the plate by a Riemann sum. Then express the force as an integral and evaluate it.

8.

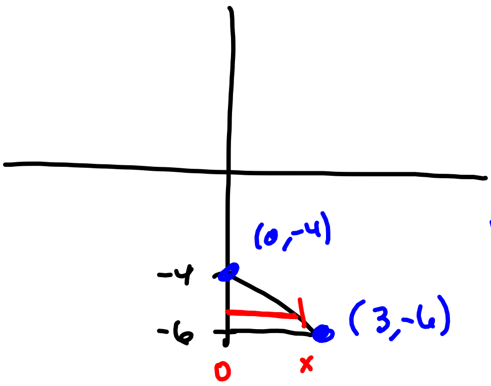


$2\int_{-6}^{-4}$

Use symmetry: Double the force on this:



$\frac{1}{2}$ of the area



$m = \frac{-6-(-4)}{3-0} = \frac{-2}{3} = m$

$y = m(x - x_1) + y_1$

$= -\frac{2}{3}(x-0) + -4$

$\Rightarrow y = -\frac{2}{3}x - 4$

Solve for x:

$3y = -2x - 12$

$-2x = 3y + 12$

$x = -\frac{3}{2}y - 6$

$2\int_{-6}^{-4}\left(-\frac{3}{2}y - 6\right) dy$

$2\int$

$\underbrace{2\left(-\frac{3}{2}y - 6\right)dy}_{\text{Area}} \quad \underbrace{(-y)}_{\text{Depth}} \frac{1000\ kg}{m^3}$

$\left(\frac{1\ g}{cm^3}\right)\frac{1\ kg}{(1000\ g)} \cdot \frac{10^6\ cm^3}{1\ m^3} = \frac{1000\ Kg}{m^3}$

$(x-h)^2 + (y-k)^2 = r^2$

$y = \pm\sqrt{r^2 - (x-h)^2} + K$

$x = \pm\sqrt{r^2 - (y-k)^2} + h$

§8.2 #26? (video) e^{-x} $x \ge 0$ about x-axis

$$2\pi \int_0^\infty e^{-x}\sqrt{1+e^{-2x}}\,dx$$

$u = e^{-x}$
$du = -e^{-x}dx$

$x = \infty$
$u = e^{-\infty} = 0$
$x = 0$
$u = e^{-0} = 1$

$$= -2\pi \int_0^\infty \sqrt{1+e^{-2x}}\,(-e^{-x}dx) = -2\pi \int_1^0 \sqrt{1+u^2}\,du$$

so,

$$+2\pi \int_0^1 \sqrt{1+u^2}\,du = \left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln\left(u+\sqrt{1+u^2}\right)\right]_0^1$$

$$= \frac{1}{2}\sqrt{1+1^2} + \frac{1}{2}\ln\left(1+\sqrt{1^2+1}\right) -$$

$$\left[\frac{0}{2} - \underbrace{+\frac{1}{2}\ln\left(0+\sqrt{0^2+1}\right)}_{\frac{1}{2}\ln(1)=0}\right]$$

$$= \boxed{\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln\left(1+\sqrt{2}\right)}$$

$f(x)$ = cont^s random probability distribution

x = random variable

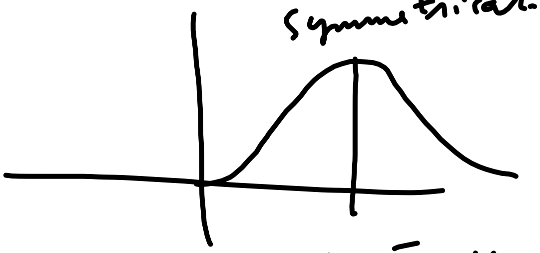
$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\text{Average} = \frac{\sum xf}{\sum f} \text{ "=" } \frac{\int_{-\infty}^{\infty} x f(x)\,dx}{\int_{-\infty}^{\infty} f(x)\,dx} = \int_{-\infty}^{\infty} x f(x)\,dx$$

Median = middle # = m,

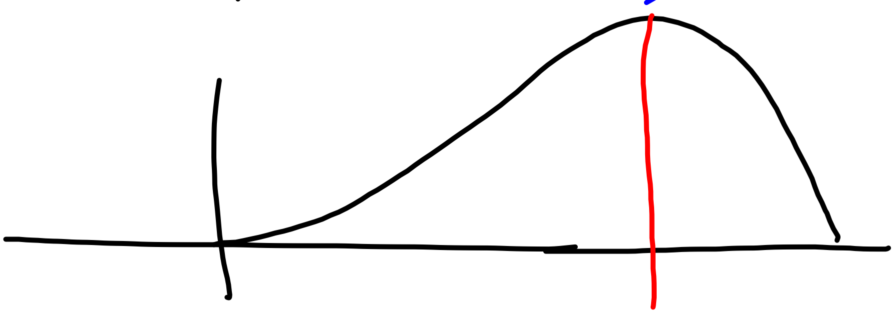
$$\int_{m}^{\infty} f(x)\,dx = \frac{1}{2}$$ & solve for m.

symmetrical.



$m = \bar{x} = \mu$

Teacher sucks



MODE

where do $\bar{x} = \mu$

& median go, Steve?

Normal Distribution:

$$\boxed{f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

σ = standard deviation.

$$\frac{\sum (x-\bar{x})^2 f}{\sum f} \text{ " = " } \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

μ = mu = $\bar{x}$ = average

§ 9.1

$\frac{df}{dx} = kf$ Exponentials are proportional to their growth rate.

$\frac{d}{dx}\left[2^x\right] = (\ln(2))\, 2^x$ $k = \ln 2$

$\frac{d}{dx}\left[e^x\right] = (\ln(e))\, e^x = e^x$ $k = 1$

$\frac{df}{dx} = f(x) \Rightarrow f(x) = e^x$

$\frac{dy}{dx} = y \Rightarrow y = e^x$

$y' = y$ $y = e^x$

$y' - y = 0$

$Dy - 1y = 0$

$(D-1)y = 0$

$D - 1 = 0 \Rightarrow$
$D = 1$ = coefficient of x in e^{kx}

$y' = 7y$

$y' - 7y = 0$

$Dy - 7y = 0$

$y\boxed{(D-7)} = 0$ Characteristic Polynomial

$D = 7$

$y = e^{7x}$

$y' = 7e^{7x}$

$y' - 7y = 0$!

$$y'' - 3y' + 2y = 0$$

$$D^2 - 3D + 2 = 0$$

$$(D-2)(D-1) = 0$$

$$D = 1, 2$$

$y = c_1 e^{x} + c_2 e^{2x}$ is the

$$\boxed{\begin{aligned} y(0) &= 1 \\ y'(0) &= 1 \end{aligned}}$$

$$y' = c_1 e^{x} + 2c_2 e^{2x}$$

$$\begin{aligned} y'(0) &= c_1 + 2c_2 = 1 \\ y(0) &= c_1 + c_2 = 1 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 1 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -1 & 0 \end{array}\right]$$

$\boxed{c_2 = 0}$?!

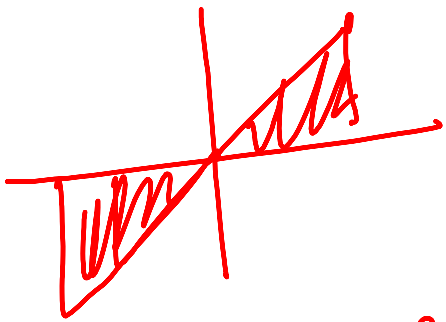
$$c_1 + 2c_2 = 1$$

$$\boxed{c_1 = 1}$$

$$\int_{-8}^{8} dy \qquad u = 64 - y^2$$

$$u = 8 \rightarrow 64 - (8)^2$$

=



$$\int_{-8}^{8} \underbrace{y}_{\text{ODD}} \sqrt{\underbrace{64-y^2}_{\text{Even}}}\, dy = 0$$

\+ always

$$\int_0^0 = 0$$

$$f(y) = y\sqrt{64-y^2}$$

$$f(-y) = (-)(+) = -f(y)$$