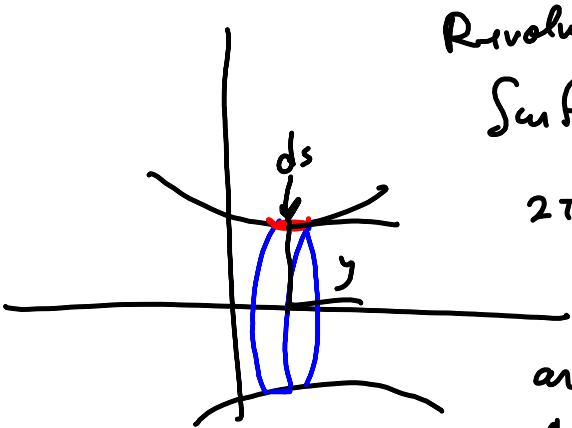


Revolving about the x-axis:

Surface area is

$$2\pi\int_a^b y\,ds$$

area of the ribbon is circumference times increment of length

If given $y=f(x)$

Then

$$2\pi\int f(x)\,ds$$

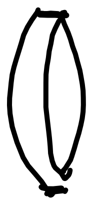
$$= 2\pi\int f(x)\sqrt{1+(f'(x))^2}\,dx$$

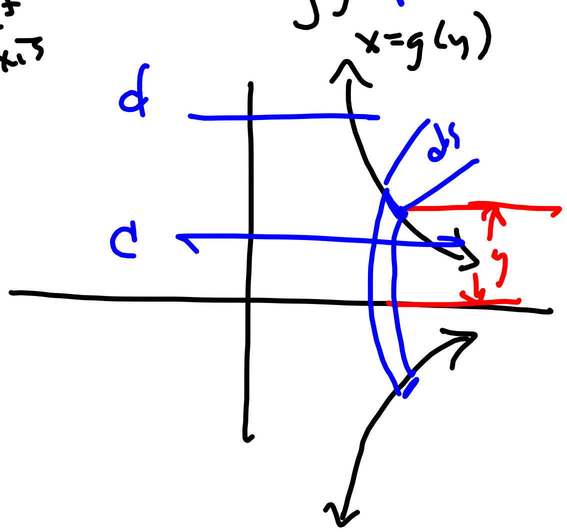
If given $x=g(y)$

$$= 2\pi\int y\sqrt{1+(g'(y))^2}\,dy$$

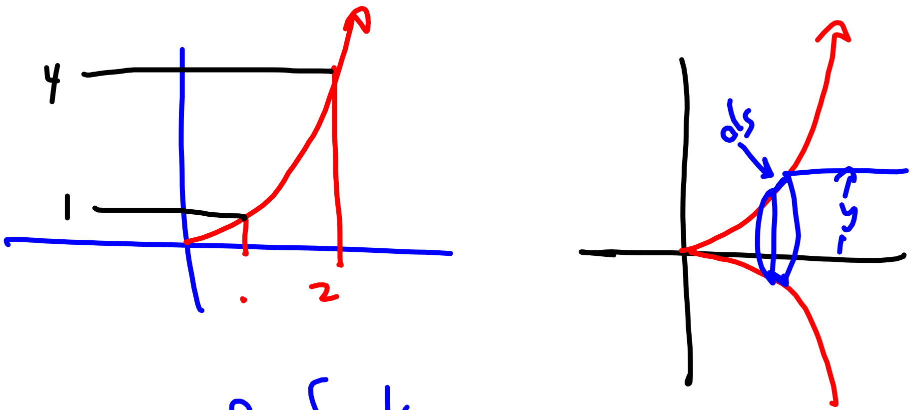
$\int 2\pi y\,ds$ About x-axis

$\int 2\pi x\,ds$ About y-axis





$f(x) = x^2 \quad x=1 \text{ to } x=2 \quad \text{about } x\text{-axis}$



$$2\pi \int y\, ds$$

$$= 2\pi \int_1^2 x^2 \sqrt{1+(2x)^2}\, dx$$

Revolve $x = \underset{=g(y)}{\sqrt{y}}$ from $y=1$ to $y=4$

$$2\pi \int_{y=1}^{y=4} y\, ds = 2\pi \int_1^4 y \sqrt{1 + \frac{1}{4y}}\, dy$$

$$x = \sqrt{y} = y^{\frac{1}{2}}$$

$$\frac{dx}{dy} = g'(y) = \frac{1}{2} y^{-\frac{1}{2}}$$

$$(g'(y))^2 = \frac{1}{4} y^{-1}$$

$$\left(y^{-\frac{1}{2}}\right)^2 = y^{-\frac{1}{2}\cdot 2} = y^{-1}$$

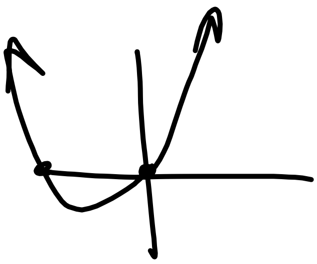
y-axis

$$2\pi \int x\, ds = 2\pi \int g(y) \sqrt{1 + g'(y)^2}\, dy$$

Given $y = f(x)$

$$2\pi \int_1 x \sqrt{1 + (f'(x))^2}\, dx$$

$$2\pi \int_0 x\sqrt{1+4x^2+4x+1}\,dx$$



$$\frac{d}{dx}(x+x^2) = 2x+1$$

$$(2x+1)^2 = 4x^2+4x+1$$

$y = x^2 + x$

Invert it

$x = g(y)$

$= -\frac{1}{2} + \sqrt{x+\frac{1}{4}}$

$y^2 + y = x$

$y^2 + y + (\frac{1}{2})^2 = x + \frac{1}{4}$

$(y+\frac{1}{2})^2 = x + \frac{1}{4}$

$y + \frac{1}{2} = \pm\sqrt{x+\frac{1}{4}}$

we're inverting the right side of the parabola

$y = -\frac{1}{2} + \sqrt{x+\frac{1}{4}}$

$y + y^3 = x$

x-axis $2\pi \int y\,ds$

y-axis $2\pi \int x\,ds$

$x(0)=0$
$x(1)=2$

$x = y + y^3$ about x- & y-axis
$y \in [0,1]$ $(x \in [0,2])$

x-axis: $2\pi \int_a^b y\,ds = 2\pi \int_0^1 y\sqrt{1+(3y^2+1)^2}\,dy$

$g(y) = y + y^3$

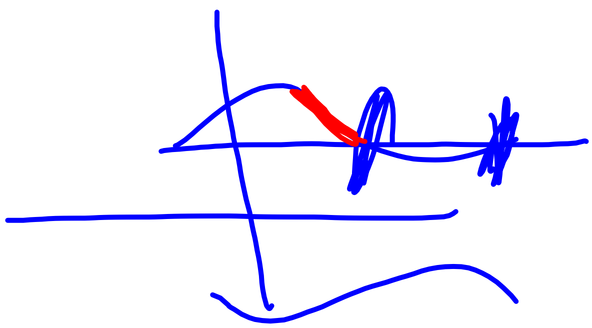
$g'(y) = 1+3y^2$

$(3y^2+1)^2 = 9y^4$

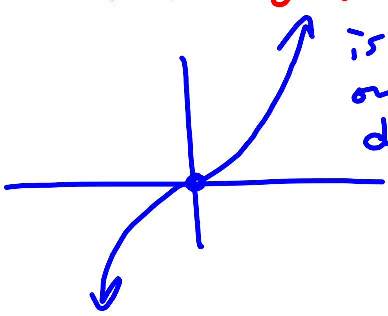
y-axis: $2\pi \int_a^b x\,ds$

$= 2\pi \int (y+y^3)\sqrt{1+(3y^2+1)^2}\,dy$

Main concern:
These functions need to be 1-to-1 to play this game.



$y+y^3 = y(y^2+1)$



is 1-to-1 on its domain.