$f(x) = y = \frac{1}{3}x^3 + \frac{1}{4x} = \frac{1}{3}x^3 + \frac{1}{4}x^{-1} \Rightarrow$

$f'(x) = \frac{3}{3}x^2 - \frac{1}{4}x^{-2} = x^2 - \frac{1}{4}x^{-2}$

$\int_1^2 \sqrt{1 + (f'(x))^2}\, dx$

$f'(x)^2 = (x^2)^2 - 2(x^2)(\frac{1}{4}x^{-2}) + (\frac{1}{4}x^{-2})^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$= x^4 - \frac{1}{2} + \frac{1}{16}x^{-4}$

$= (x^2 - \frac{1}{4}x^{-2})^2$

$(\frac{1}{4})^2 (x^{-2})^2 = \frac{1^2}{4^2}(x^{-2\cdot 2})$

$\int_1^2 \sqrt{1 + x^4 + \frac{1}{2} + \frac{1}{16}x^{-4}}\, dx$

$= \int_1^2 \sqrt{x^4 + \frac{3}{2} + \frac{1}{16}x^{-4}}\, dx = \int_1^2 x^{-2}\sqrt{(x^4 + \frac{3}{4})^2 - \frac{1}{2}}\, dx$

$x^4 + \frac{3}{2} + \frac{1}{16}x^{-4} = x^{-4}\left(x^8 + \frac{3}{2}x^4 + \frac{1}{16}\right)$

$u^2 + \frac{3}{2}u + \frac{1}{16}$

$x^8 = u^2$

$x^4 = u$

$u^2 + \frac{3}{2}u + (\frac{3}{4})^2 - \frac{9}{16} + \frac{1}{16}$

$= (u + \frac{3}{4})^2 - \frac{1}{2}$

$= (x^4 + \frac{3}{4})^2 - \frac{1}{2}$

$x^{-2}\sqrt{(x^4 + \frac{3}{4})^2 - \frac{1}{2}} = \sqrt{x^{-4}(x^4 + \frac{3}{4})^2 - \frac{1}{2}}$

$= (x^2 + \frac{3}{4}x^{-2})^2 - \frac{1}{2}$

$x^{-4} = (x^{-2})^2$

$(x^{-2})^2(x^4 + \frac{3}{4})^2$

$= ((x^{-2})(x^4 + \frac{3}{4}))^2$

$= (x^2 + \frac{3}{4}x^{-2})^2$

$$y = \frac{1}{3}x^3 + \frac{1}{4x} = \frac{1}{3}x^3 + \frac{1}{4}x^{-1}$$

$$y' = x^2 - \frac{1}{4}x^{-2}$$

$$(y')^2 = (x^2)^2 - 2(x^2)\left(\frac{1}{4}x^{-2}\right) + \left(\frac{1}{4}x^{-2}\right)^2 \qquad (y')^2 = \left(x^2 - \frac{1}{4}x^{-2}\right)^2$$

$$= \boxed{x^4 - \frac{1}{2} + \frac{1}{16}x^{-4}}$$

$$1 + (y')^2 = 1 + x^4 - \frac{1}{2} + \frac{1}{16}x^{-4}$$

$$= \boxed{x^4 + \frac{1}{2} + \frac{1}{16}x^{-4}}$$

$$= \left(x^2 + \frac{1}{4}x^{-2}\right)^2$$

So $\displaystyle\int_1^2 \sqrt{\left(x^2 + \frac{1}{4}x^{-2}\right)^2}\,dx = \int_1^2 \left|x^2 + \frac{1}{4}x^{-2}\right| dx$

$$= \int_1^2 \left(x^2 + \frac{1}{4}x^{-2}\right) dx = \left[\frac{1}{3}x^3 - \frac{1}{4}x^{-1}\right]_1^2$$

$$= \frac{1}{3}(2)^3 - \frac{1}{4}(2)^{-1} - \left(\frac{1}{3}(1)^3 - \frac{1}{4}(1)^{-1}\right)$$

$$= \frac{8}{3} - \frac{1}{4}\cdot\frac{1}{2} - \left(\frac{1}{3} - \frac{1}{4}\right)$$

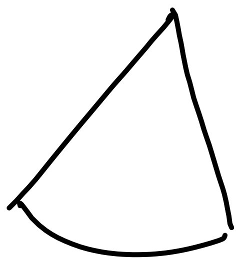
$$= \frac{8}{3} - \frac{1}{8} - \frac{1}{12} \qquad \text{LCD} = 2\cdot2\cdot2\cdot3 = 24$$

$3 \quad 2\cdot2\cdot2 \quad 2\cdot2\cdot3$

$$= \frac{8}{3}\cdot\frac{8}{8} - \frac{1}{8}\cdot\frac{3}{3} - \frac{1}{12}\cdot\frac{2}{2}$$

$$= \frac{64-3-2}{24} = \frac{59}{24}, \text{ obviously.}$$

$$\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$



$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

<u>EXCEPT</u> when $n = -1$

The power rule exception

$$\int x^{-1} dx = \int \frac{dx}{x} = \ln|x| + C$$

Sort of
(Jury nullification)