S6.8 #58

$$\lim_{x\to 0^+} \left(\tan(2x)\right)^x = 0^0 \text{ situation}$$

$$y = \left(\tan(2x)\right)^x$$

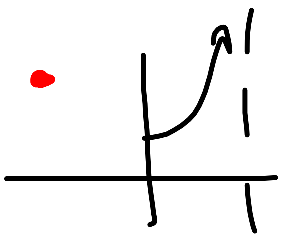
$$\ln(y) = x \ln(\tan(2x)) \xrightarrow{x\to 0^+} 0\cdot(-\infty)$$

$$= \begin{cases} \nearrow \dfrac{0}{0} \\ \searrow \dfrac{-\infty}{\frac{1}{0}} = \dfrac{-\infty}{\infty} \end{cases}$$

Either way, L'Hopital sitch.

$$x \ln(\tan(2x)) = \frac{\ln(\tan(2x))}{\frac{1}{x}} \xrightarrow{x\to 0^+} \frac{-\infty}{\infty}$$

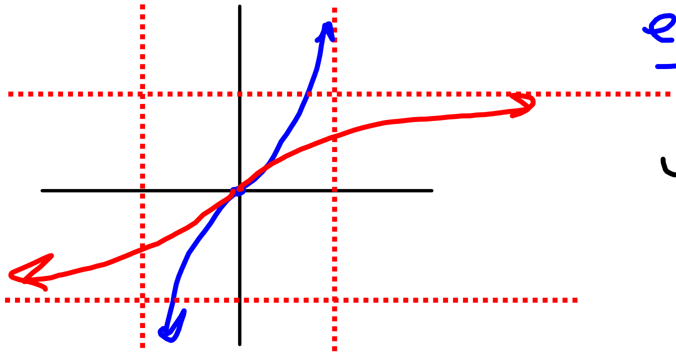
$$\xrightarrow[x\to 0^+]{L'H} \frac{2\sec^2(2x)}{-\frac{1}{x^2}} \xrightarrow{x\to 0^+} \frac{1}{-\infty} \boxed{= 0}$$



$x = \frac{\pi}{2}$

$$(fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

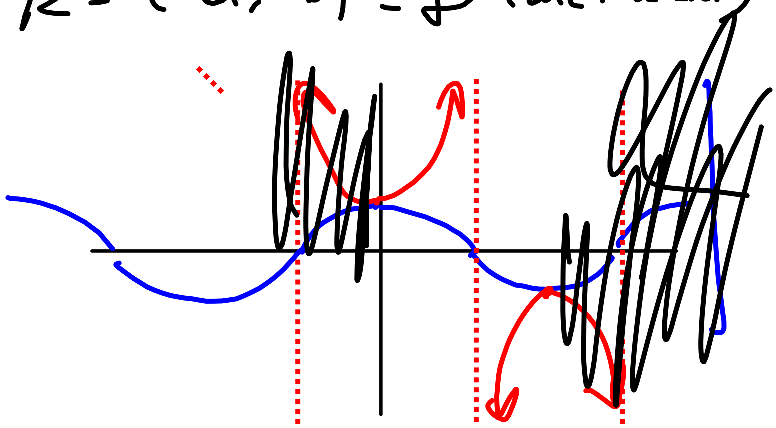


$$\frac{e^x - e^{-x}}{2}$$

$y' =$

$\mathcal{D} = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = \mathcal{R}(\arctan(x))$

$\mathcal{R} = (-\infty, \infty) = \mathcal{D}(\arctan(x))$



$\mathcal{D}(\sec) = [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

$= \mathcal{R}(\operatorname{arcsec}(x))$

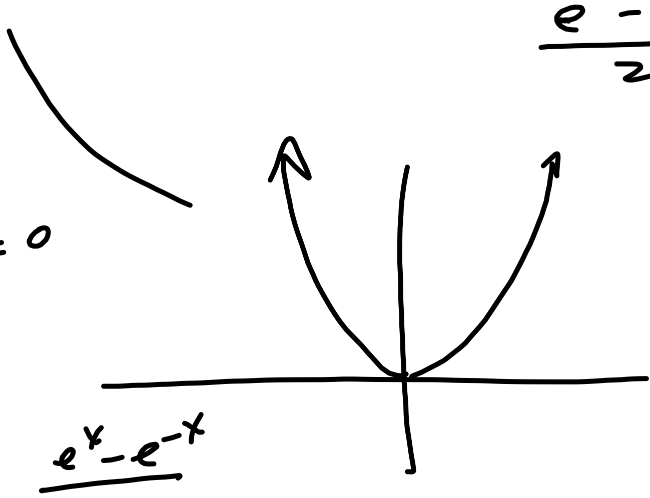
$$\boxed{\begin{aligned}\mathcal{R}(\sec) &= [1, \infty) \cup (-\infty, -1] \\ &= (-\infty, -1] \cup [1, \infty)\end{aligned}}$$

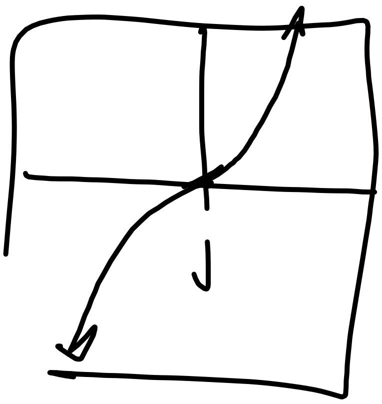
$$y = \sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

$$y' = \cosh(x) = \frac{e^{x} + e^{-x}}{2} > 0 \quad \forall x$$

$$\frac{e^{0} - e^{-0}}{2} = \frac{1 - \frac{1}{1}}{2} = 0$$

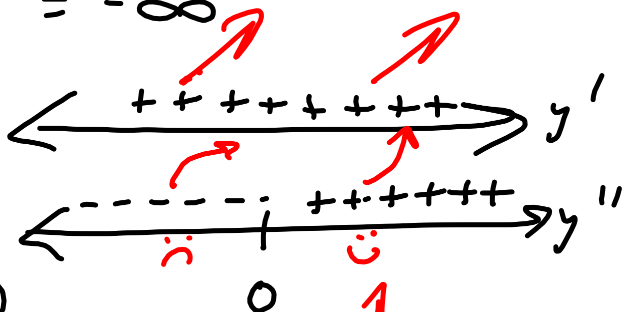
$$\frac{e^{x} - e^{-x}}{2} = 0$$





$$\lim_{x \to -\infty} \frac{e^{x} - e^{-x}}{2} = \frac{0 - \infty}{2} = -\infty$$

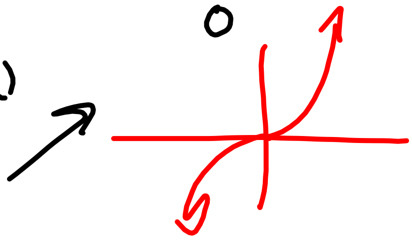
$y'' = \sinh(x)$



$y = \sinh(x)$

$y' = \cosh(x)$

$y'' = \sinh(x)$



$$\lim_{x\to 1^+} \ln(x^7-1) - \ln(x^5-1)$$

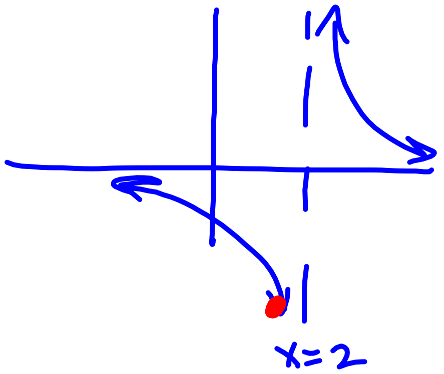
$$\frac{7x^6(x^5-1) - (x^7-1)(5x^4)}{(x^5-1)^2}$$

$$= \lim_{x\to 1^+} \ln\left(\frac{x^7-1}{x^5-1}\right)$$

$$\lim_{x\to 1^+} \frac{x^7-1}{x^5-1} \overset{L'H}{=} \lim_{x\to 1^+} \frac{7x^6}{5x^4} = \lim_{x\to 1^+} \frac{7}{5}x^2 = \frac{7}{5}$$

$$= \lim_{u\to \frac{7}{5}^+} \ln(u) = \boxed{\ln\left(\frac{7}{5}\right)}$$

$$\lim_{x\to 2^+} e^{\frac{1}{x-2}} = \lim_{u\to \infty^-} e^{u} = \infty$$



$$\lim_{x\to 2^-} e^{\frac{1}{x-2}} = \lim_{u\to -\infty} e^{u} = 0$$

$$e^{-\text{BIG}} = \frac{1}{e^{\text{BIG}}} = \text{Small}$$