6.8 #4b

$\lim \quad f(x)^{g(x)} \quad f(x) = 0 \;\&\; g(x) = \infty$

$y = f(x)^{g(x)} = 0$

$\ln(y) = g(x)\ln(f(x)) \xrightarrow{x \to \infty} \infty \cdot \underline{\ln(0)}$

$\downarrow$

$-\infty\,?$

$\lim_{x\to a} f(x)^{g(x)} \text{ "=" } 0^{\infty}$

Need to think about how it gets there...

$\ln(y) \text{ "=" } \infty \cdot (-\infty) = -\infty$

$y = e^{-\infty} \text{ "=" } 0$

§6.8 #41

$$\lim_{x\to 0} \frac{\cos(x) - 1 + \frac{1}{2}x^2}{x^4} \text{ "=" } \frac{0}{0}$$

$$\frac{\cos(x-1) + \frac{1}{2}x^2}{x^4} \xrightarrow{x\to 0} \frac{\cos(-1)}{0} \quad \cancel{\nexists}$$

$$\frac{\cos(x) - 1 + \frac{1}{2}x^2}{x^4} \xrightarrow[L'H]{x\to 0} \frac{-\sin x + x}{4x^3}$$

$$\xrightarrow[L'H]{x\to 0} \frac{-\cos x + 1}{12x^2} \xrightarrow[L'H]{x\to 0} \boxed{\frac{\sin x}{24x}} \xrightarrow{x\to 0} \frac{1}{24}$$

$\lim (cf) = c \lim f$ if $\underline{c \in \mathbb{R}}$ & $\lim f$ $\exists$.

Recall in proof of $\frac{d}{dx}[\sin x] = \cos x$, in CALC I, we proved $\lim_{x\to 0} \frac{\sin x}{x} = 1$

$$\xrightarrow[L'H]{x\to 0} \frac{\cos(x)}{24} \xrightarrow{x\to 0} \frac{1}{24}$$

$\lim_{x\to 0} \frac{\sin(3x)}{x} = 3$, so that means $\sin(3x) = 3\sin(x)$?!

No!

$$\lim_{x\to 0} \frac{\sin(3x)}{x} = 3\lim_{x\to 0} \frac{\sin(3x)}{3x} = 3\cdot 1$$