$\sinh(x) = \dfrac{e^x - e^{-x}}{2}$ $\sin(x) = \dfrac{e^{ix} - e^{-ix}}{2i}$

$\cosh(x) = \dfrac{e^x + e^{-x}}{2}$ $\cos(x) = \dfrac{e^{ix} + e^{-ix}}{2}$

$\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$

---

§6.8 L'Hospital's rule.

If $\dfrac{f(x)}{g(x)} \xrightarrow{x \to a} \dfrac{b}{b} = 1$

Then $\lim \dfrac{f(x)}{g(x)} = \lim \dfrac{f'(x)}{g'(x)}$

Essential for stuff like

0^∞, 1^∞, $\dfrac{0}{0}$, $\dfrac{\infty}{\infty}$ situations

$$\frac{d}{dx}[\sin(x)] = \cos(x)$$

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} =$$

$$= \lim_{h \to 0} \frac{\sin x \cosh + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x)(\cos(h) - 1) + \sin(h)\cos(x)}{h}$$

$$= \lim_{h \to 0} \sin(x)\left[\frac{\cos(h) - 1}{h}\right] + \lim_{h \to 0} \frac{\cos(x)\sin(h)}{h}$$

$$" = " \sin(x)\left(\frac{0}{0}\right) + \frac{\cos(x)\left(\frac{\sin(0)}{0}\right)}{\cos(x)\left(\frac{0}{0}\right)}$$

L'Hopital says:

$$\lim_{h\to 0} \sin(x)\left[\frac{\cos(h)-1}{h}\right] = \sin(x)\lim_{h\to 0}\left[\frac{-\sin(h)}{1}\right]$$

$\frac{0}{0}$ sitch

$$= \sin(x)\cdot 0 = 0$$

$$\cos(x)\lim_{h\to 0}\frac{\sin(h)}{h} = \cos(x)\lim_{h\to 0}\frac{\cos(h)}{1} = \cos(x)\cdot 1 = \cos(x)$$

$$\lim_{x \to 1^+} x^{\frac{1}{1-x}} \text{ "=" } 1^{\infty} \ ?$$

$$y = x^{\frac{1}{1-x}}$$

$$\ln(y) = \frac{1}{1-x} \ln(x) \xrightarrow{x \to 1^+} \frac{0}{0} \text{ is indeterminate}$$

$$\lim_{x \to 1^+} \ln(y) = \lim_{x \to 1^+} \frac{\ln(x)}{1-x} \overset{l'H}{=} \lim_{x \to 1^+} \frac{\frac{1}{x}}{-1}$$

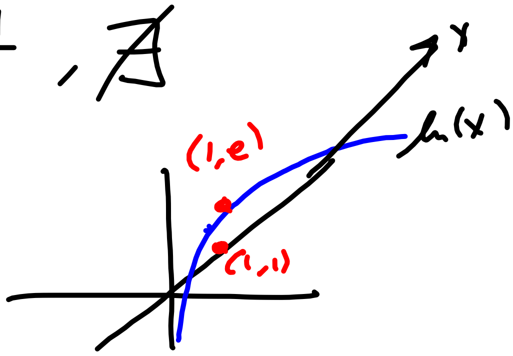
$$\lim_{x \to 1^+} \ln(y) = -1$$

$$\lim_{x \to 1^+} e^{\ln(y)} = \boxed{e^{-1} = \lim_{x \to 1^+} x^{\frac{1}{1-x}}}$$

$$\lim_{x \to \infty} x^{\frac{1}{x}}$$

$\lim_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{e^{x-1}}\right)$ "=" $\infty - \frac{1}{e}$, $\nexists$

$= \infty$



$$\lim_{x\to\infty}\left(\frac{(\ln(x))^2}{x}\right) \text{ "=" } \frac{\infty}{\infty}$$

$$\lim_{x\to\infty}\frac{(\ln(x))^2}{x} \overset{L'H}{=} \lim_{x\to\infty}\frac{2(\ln(x))\frac{1}{x}}{1} \boxed{= \lim_{x\to\infty}\frac{2\ln(x)}{x}} \text{ "=" } \frac{\infty}{\infty}$$

$$\overset{L'H}{=} \lim_{x\to\infty}\frac{2\left(\frac{1}{x}\right)}{1} = \lim_{x\to\infty}\frac{2}{x} = 0$$

§6.6 #66

$$\int \frac{dx}{x\sqrt{x^2 - 4}} = \sec^{-1}(x) + C$$

$$\frac{d}{dx}\left(\sec^{-1}(x)\right) = \frac{1}{x\sqrt{x^2 - 1}}$$

$$\int \frac{dx}{x\sqrt{x^2 - 4}} = \int \frac{dx}{x\sqrt{4\left(\frac{x^2}{4} - 1\right)}}$$

$$= \int \frac{dx}{2x\sqrt{\frac{x^2}{4} - 1}} = \int \frac{dx}{2x\sqrt{\left(\frac{x}{2}\right)^2 - 1}}$$

$u = \frac{x}{2} \Rightarrow x = 2u$

$du = \frac{dx}{2} \Rightarrow dx = 2du$

$$= \int \frac{2\,du}{2(2u)\sqrt{u^2 - 1}}$$

cos(u)? arcsec(u)?

$\frac{1}{2}\,\mathrm{arcsec}\left(\frac{x}{2}\right) + C$

$$= \frac{1}{2}\int \frac{du}{u\sqrt{u^2 - 1}} = \frac{1}{2}\boxed{\sec^{-1}(u)} + C$$

$$= \boxed{\frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right) + C}$$

Fixes my error on soln's.