§6.3 # 68

$f(x) = e^{-x}$

perp to $2x - y = 8$

$2x - y = 8$

$-y = 8 - 2x$

$y = 2x - 8$

$m = 2$ want $\perp$ $\Rightarrow$ want $m_\perp = -\frac{1}{2}$

$f'(x) = -e^{-x} \overset{SET}{=} -\frac{1}{2} \Rightarrow$

$e^{-x} = \frac{1}{2} \Rightarrow$

$-x = \ln\left(\frac{1}{2}\right) \Rightarrow$

$x = -\ln\left(\frac{1}{2}\right) = \boxed{\ln(2) = x_1}$

$f(\ln(2)) = e^{-\ln(2)} = e^{\ln\left(\frac{1}{2}\right)} = \boxed{\frac{1}{2} = y_1}$

$L(x)$ = Linearization of f @ x

$y - y_1 = m(x - x_1)$

= Tangent line

$y = m(x - x_1) + y_1$

$= f'(x_1)(x - x_1) + y_1$

$$\boxed{y = -\frac{1}{2}(x - \ln(2)) + \frac{1}{2}}$$

POIFECK

$-\frac{1}{2}\left(x - \left(-\ln\left(\frac{1}{2}\right)\right)\right) + \frac{1}{2}$

(22) $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = f(v)$

solve for v.

My way:

$\dfrac{m_0}{\sqrt{1-\frac{m^2}{c^2}}} = v$ & solve for m.

Book way: Just solve 1st eq'n for v.

Then swap m & v.

$m\sqrt{1-\frac{v^2}{c^2}} = m_0$

$m^2\left(1-\frac{v^2}{c^2}\right) = m_0^2$

$m^2 - \frac{m^2v^2}{c^2} = m_0^2$

$-\frac{m^2v^2}{c^2} = m_0^2 - m^2$

$v^2 = -\frac{c^2}{m^2}\left(m_0^2 - m^2\right) = \frac{c^2}{m^2}\left(m^2 - m_0^2\right)$

$v = \pm \frac{c}{m}\sqrt{m^2 - m_0^2}$

Take positive

$v = \frac{c}{m}\sqrt{m^2 - m_0^2}$

§6.3 #66

$f' > 0$ inc. $\quad f'' > 0$ ☺

$f' < 0$ dec. $\quad f'' < 0$ ☹

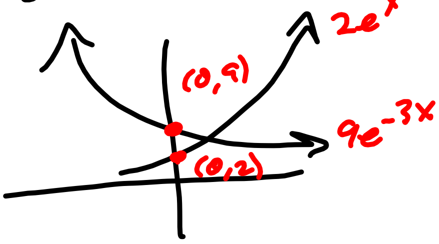
$y = 2e^{x} - e^{-3x}$ is concave down where?

want $y'' < 0$

$y' = 2e^{x} + 3e^{-3x}$

$$\boxed{y'' = 2e^{x} - 9e^{-3x}} \overset{\text{SET}}{=} 0$$

eventually positive



$= e^{-3x}\left[2e^{4x} - 9\right] = 0$

$\Rightarrow e^{4x} = \frac{9}{2}$

$4x = \ln\left(\frac{9}{2}\right)$

$x = \frac{1}{4}\ln\left(\frac{9}{2}\right)$

$-$ $\qquad$ $+$ $\quad f''$

$\frac{\ln(9/2)}{4}$

So C. Down on $\left(-\infty, \frac{\ln(9/2)}{4}\right)$

$$\frac{d}{dx}\left[\ln(f(x))\right] = \frac{f'(x)}{f(x)}$$

$$\frac{d}{dx}[y] = \frac{y'}{y}$$

Logarithmic Differentiation can speed things up:

$$\frac{d}{dx}\left[\sqrt[3]{(x^2+1)^5} \Big/ \sqrt[5]{(x^2-5x)^3}\right] \qquad \ln\left((x^2-5x)^{\frac{3}{5}}\right)$$

$\ln y = \ln($ mess $)$

$$\ln y = \frac{5}{3}\ln(x^2+1) - \frac{3}{5}\ln(x^2-5x)$$

$$\frac{y'}{y} = \frac{5}{3}\left(\frac{2x}{x^2+1}\right) - \frac{3}{5}\left(\frac{2x-5}{x^2-5x}\right)$$

$$y' = \left[\frac{5}{3}\left(\frac{2x}{x^2+1}\right) - \frac{3}{5}\left(\frac{2x-5}{x^2-5x}\right)\right]\frac{\sqrt[3]{(x^2+1)^5}}{\sqrt[5]{(x^2-5x)^3}}$$

is essential for

$y = f(x)^{g(x)}$

we don't have any other technique for handling variable function to variable power.

$$\left(2x^2-5x\right)^{7x^2+19}$$

$$\ln y = g(x)\ln(f(x))$$

$$\frac{y'}{y} = g'\ln f + g\frac{f'}{f}$$

$$y' = \left(g'\ln f + \frac{gf'}{f}\right)y$$